

## A33 Reservoir Management Using Two-stage Optimization with Streamline Simulation

T. Wen\* (Stanford University), M.R. Thiele (Streamsim Technologies/ Stanford University), D. Echeverría Ciaurri (IBM Thomas J. Watson Research Center), K. Aziz (Stanford University) & Y. Ye (Stanford University)

# SUMMARY

Waterflooding is a common secondary oil recovery process. Performance of waterfloods in mature fields with a significant number of wells can be improved with minimal infrastructure investment by optimizing injection/production rates of individual wells. However, a major bottleneck in the optimization framework is the large number of reservoir flow simulations often required. In this work we propose a new method based on streamline-derived information that significantly reduces these computational costs in addition to making use of the computational efficiency of streamline simulation itself. We seek to maximize the long-term net present value of a waterflood by determining optimal individual well rates, given an expected albeit uncertain oil price and a total fluid injection volume. We approach the optimization problem by decomposing it into two stages which can be implemented in a computationally efficient manner. The two-stage streamline-based optimization approach can be an effective technique when applied to reservoirs with a large number of wells in need of an efficient waterflooding strategy over a 5 to 15 year period.



## Introduction

Waterflooding is a common secondary oil recovery process in which water is injected into an oil bearing reservoir using strategically placed injectors so as to maintain pressure and sweep oil to adjacent production wells. However, the efficiency of a waterflood will depend on a number of factors such as the differences in fluid properties, the spatial distribution of rock properties such as permeability, porosity and rock type, as well as the injection/production rates imposed at the wells.

Many waterfloods are considered 'brown fields' meaning that production has been occurring for many years (>10), oil is produced in conjunction with large volumes of water (>75%), and the total number of wells is generally high (>100). However, despite the high water production, a significant amount of by-passed oil usually remains in such floods because they have generally been operated at sub-optimal conditions for many years. As a result, oil production may be enhanced significantly with minimal infrastructure investment simply by changing injection/production rates of individual wells currently operating. Additional enhancements, such as re-completions, side-tracks, and new infill wells can increase recovery further, although these interventions tend to be more capital intensive. The effort to improve the performance of brown fields has lead to closed-loop reservoir management strategies in which the inherent nonlinearity of the recovery processes is addressed formally by means of optimization approaches (Jansen et al. (2009); Wang et al. (2009); Sarma et al. (2005)).

In the context of closed-loop reservoir management, production optimization is often formulated assuming a given geological scenario and recovery mechanism (waterflood, CO<sub>2</sub>-flooding, etc.) The goal is to maximize/minimize an objective function (e.g. cumulative oil production, NPV, total water production) under a set of physical constraints (e.g. available water supply, individual well production/injection capabilities), and economic constraints (e.g. oil price, tax regimes). The most immediate control variables are injection/production well rates as these represent how floods are actually managed/implemented. Setting optimal well target rates is the focus of our work.

The main challenge in formulating the production optimization problem is to account for the nonlinearity inherent in the recovery process. Nonlinear optimization methods can be roughly divided into two main categories. The first family of methods refers to those procedures that use derivative information determined from the cost function and/or constraints. In general, gradient-based techniques (Nocedal and Wright (2006); Luenberger and Ye (2008)) converge to local optima. Usually these solutions may not be acceptable from a cost function perspective if important nonlinearities are present in the optimization. The most straightforward approach for computing derivatives is by means of finite-difference approximations. In this case, the estimation of the gradients requires a number of cost function evaluations on the order of the number of control variables. Since the evaluation of the cost function usually involves complex and time-demanding simulations, approximating derivatives numerically is expensive. Additionally, a common issue in finite-difference approximations is finding the proper perturbation size in the numerical gradient used (large perturbation sizes may yield inaccurate derivatives, and small perturbation sizes may cause numerical issues due to the resolution of the simulator). In some cases gradient information can be extracted from the simulator in an efficient manner; adjoint-based methods (Sarma et al. (2006); Brouwer and Jansen (2002)) are a very well-known technique for rapid computation of derivatives. However, these methods require detailed knowledge of and access to the source code of the simulator used, and that can be a limitation in practice.

The second family of optimization techniques comprises procedures that do not require gradient information. Derivative-free algorithms (Kolda et al. (2003); Conn et al. (2009); Kramer et al. (2011); Echeverría Ciaurri et al. (2011)) can be subdivided into local optimization methods and global search schemes. The first type of methods have computational costs similar to gradient-based methods, but are somewhat more robust from a theoretical perspective (for example, the issue of the perturbation size for the finite differences is solved in many derivative-free algorithms). Although (most) local derivative-free meth-



ods guarantee convergence only to local optima, many of these techniques incorporate some amount of global exploration that may avoid being trapped in solutions that are not satisfactory from a cost function point of view. Examples of derivative-free optimization methods that rely on local search are Generalized Pattern Search (GPS; Audet and Dennis Jr (2002)), Mesh Adaptive Direct Search (MADS; Audet and Dennis Jr (2006)), and Hooke-Jeeves Direct Search (HJDS; Hooke and Jeeves (1961)). In global search schemes the optimization space is analyzed much more thoroughly than in local methods, but at the expense of a larger computational cost. It is important to note that in the majority of practical optimization problems the global optimum cannot be obtained (the *curse of dimensionality* makes this enterprise infeasible as soon as the number of optimization variables is larger than a few tens, something that happens very often in real-life situations). Many global search algorithms resort to stochastic heuristics to prevent the whole process from terminating prematurely at a solution with a cost function that is unacceptable. These heuristics are rarely supported by formal optimization theory, and this is translated in algorithmic parameters that are difficult to tune (as is the case of the population size in genetic algorithms). As a consequence, the use of global search methods requires a significant amount of experience, and in some cases the performance of these methods can be rather unpredictable. Examples of global search procedures are genetic algorithms (GAs; Goldberg (1989)), particle swarm optimization (PSO; Kennedy and Eberhart (1995)), and differential evolution (DE; Storn and Price (1997)). We note that derivative-free optimization methods can be easily implemented in a distributed manner, and therefore be fairly efficient in terms of elapsed clock time (if a parallel computing environment is available).

In our work, we seek to maximize the long-term (generally 5 to 15 years) NPV of a waterflood, given a cumulative target limit of the total field injection volume in conjunction with an uncertain oil price. The goal is to determine (near) optimal individual well rates that are sustainable (subject to constraints, such as gradual changes over time as well as globally maximum/minimum total fluid rate handling capacities). To reduce the computational costs, we make use of streamline-derived information to drive the well rate changes while also taking advantage of the computational efficiency of streamline simulation itself, which is generally well-suited for modeling waterfloods. We additionally speed-up the entire optimization process by decomposing the optimization problem into two stages: short-term and long-term. In the short-term stage, the NPV is maximized by setting optimal well controls (injected/produced total fluid well rates) subject to a fixed total volume injection target. In the long-term stage, the sequence of total volume injection targets of all short-term periods is iteratively updated according to the expected oil price and the long-term reservoir behavior approximated by an exponential decline model.

This two-stage decomposition is computationally efficient because the optimizations for the short-term periods use local information, that can be interpreted as approximate derivatives, derived from the connectivity information provided by the streamlines which require only a single streamline simulation (and thus, the computational cost associated to the estimation of the approximate derivatives becomes independent of the total number of wells). The outer loop (long-term) optimization is accelerated by approximating the long-term reservoir behavior through an analytical (exponential) decline model. The approach is validated on two waterflood scenarios where we find a significant speedup compared to other single-stage optimization methods, for the same quality of the solution.

The paper is organized as follows. In the next section, we define the short-term optimization problem and show how streamline-based simulation can be used within the solution approach. In the section on long-term reservoir management, we define the long-term optimization problem, and explain how the exponential decline model can be combined together with streamline simulation to approximate a global, long-term optimal solution. After that, we include a section where we present example cases with practical relevance and compare our approach with a single-stage direct search (derivative-free) method. We end the paper with some discussion and conclusion.



#### Short-Term Reservoir Management

In a short-term (less than one year) reservoir management problem, we are interested in finding optimal well control settings so as to maximize profit but without accounting for fluctuations and discounting in the oil price as we assume the price change and discount factor is small in a short-term period. Before we introduce the mathematical formulation of the short-term reservoir management problem, we introduce the notation used in this section. The numbers of producers and injectors are denoted by  $N_P$  and  $N_I$ , respectively.  $(q_o^P)_j$  and  $(q_w^P)_j$  denote the oil and water rates at the *j*th producer, and the total fluid rates on the *i*<sup>th</sup> injector and the *j*<sup>th</sup> producer are respectively represented by  $(q_w^I)_i$  and  $(q_f^P)_j$ . Note that  $\mathbf{q}_w^I, \mathbf{q}_f^P$  are the vectors of the flow rates of all injectors/producers, which are the optimization variables. And  $\mathbf{q}_o^P$  is the vector of oil rates of all producers.  $r^o$  is the oil price per unit volume of oil, and  $c^{wi}$  and  $c^{wp}$  are the length of the short-term optimization period.

We define the short-term optimization problem as follows:

$$\underset{\mathbf{q}_{\mathbf{w}}^{\mathbf{I}}, \mathbf{q}_{\mathbf{f}}^{\mathbf{P}}}{\text{simize}} \qquad \left( r^{o} \sum_{j=1}^{N_{P}} \left( q_{o}^{P} \right)_{j} - c^{wp} \sum_{j=1}^{N_{P}} \left( q_{w}^{P} \right)_{j} - c^{wi} \sum_{i=1}^{N_{I}} \left( q_{w}^{I} \right)_{i} \right) \Delta t,$$
 (1a)

subject to

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$$\mathscr{F}\left(\mathbf{q}_{w}^{\mathbf{I}},\mathbf{q}_{f}^{\mathbf{P}};\mathbf{x}\right) = \mathbf{q}_{o}^{\mathbf{P}},\tag{1b}$$

$$\sum_{i=1}^{N_I} \left( q_w^I \right)_i = u, \quad \sum_{j=1}^{N_P} \left( q_f^P \right)_j = Cu, \tag{1c}$$

$$(q_o^P)_j + (q_w^P)_j = (q_f^P)_j, \quad j = 1, 2, \dots, N_P,$$
 (1d)

$$L_i^I \le \left(q_w^I\right)_i \le U_i^I, \quad i = 1, 2, \dots, N_I, \tag{1e}$$

$$L_j^P \le (q_f^P)_j \le U_j^P, \quad j = 1, 2, \dots, N_P.$$
 (1f)

In the objective function (1a), we define profit as the revenue from oil production subtracted by the cost for water injection and for the disposal/separation of produced water. We reiterate that the optimization variables are the total fluid rates on injectors and producers, i.e.  $q_w^I$  and  $q_f^P$ . The relation between the oil produced at each well and the fluid rate controls (given the current reservoir state  $\mathbf{x}$ ) is expressed in Equation (1b). The function  $\mathscr{F}$  is generally complex and highly nonlinear, and must be evaluated with a numerical reservoir simulator. In our work,  $\mathscr{F}$  is based on a streamline simulator (Streamsim Technologies (2012)) that in the examples studied here yields significant speedup with respect to finitedifference/finite-volume simulation approaches. The constraint (1c) refers to a total injection volume, here u and C are parameters given by the user and they represent the field total injection target and the voidage replacement ratio. As indicated above, the short-term optimization problem will be the basic building block used to solve the long-term optimization problem. Note that the short-term optimization problem is stated for a relatively short production time frame, and that the well flow rates are assumed constant during the period. The long-term optimization involves time explicitly, and it is essentially a loop over short-term optimization with different values of total field injection u. We will see in the section on long-term reservoir management that the sequence of u is selected so that the cumulated long-term profit determined by a fast-to-evaluate modified exponential decline model is maximized (this strategy takes into account the variation in the oil price, and avoids, in particular, low/high oil production during periods where the price is expected to be high/low). The constraint (1d) restricts that, at any producer, the sum of the oil flow rate and the water flow rate has to be equal to the total fluid rate. The fluid rate targets of each well are required to stay within a minimum/maximum range (see (1e) and (1f)).

In this work we opt for solving the optimization problem (1a)-(1f) using a derivative-free approach, motivated by the following two facts: first, the simulator available does not include methods for determining derivatives rapidly (e.g. adjoint-based procedures); second, as indicated earlier, numerical derivatives approximated by finite-differences present the problematic issue of finding the right perturbation size.



Since the computational cost associated to each evaluation of  $\mathscr{F}$  dominates all other calculations in the optimization problem, the algorithm considered for solving this problem should make economic use of the function  $\mathscr{F}$ .

Here we propose to exploit the physical interpretation of streamlines to reduce the number of evaluations of the function  $\mathscr{F}$  in the complete optimization process. In essence, by means of streamline simulation we will be able to (roughly) approximate the gradient of the cost function using only one reservoir simulation. This is a significant reduction with respect to, for example, the approximation performed via finite differences, where the number of cost function evaluations is on the order of optimization variables (and in this case equal to the number of wells). Although the gradient estimation obtained using streamlines in general will not be as good as the one determined with finite differences, in many cases (as is illustrated in later section) it will be good enough to iteratively provide improvement in the optimization, and to eventually reach the proximity of a satisfactory local solution in an efficient manner. We introduce the general concept of streamline simulation next.

#### Streamline Simulation and Flux Pattern Map

Streamline simulation is an alternative reservoir simulation approach to the more widely used finitedifference/finite-volume approaches (Aziz and Settari (1979)). Although streamlines have been in the petroleum literature since the early days of Muskat and Wyckoff (1937), modern streamline simulation emerged in the early 90s and distinguishes itself by six key ideas: 1) tracing 3D streamlines using the concept of time-of-flight (TOF) rather than arc length; (2) expressing the mass conservation equations in terms of TOF; (3) periodic updating of the streamlines in time; (4) solving the transport problems numerically along the streamlines rather than analytically; (5) accounting for gravity effects; and (6) extension to compressible flow (Thiele et al. (2010)). Because streamlines connect injectors and producers, streamline simulation has been successful at modeling floods that are dominated by the relative location and strength of injectors and producers and by the connectivity inherent in the underlying geological model. In such cases, streamline simulation is efficient in supplying engineering data that allows a more physically-based optimization approach (Batycky et al. (1997); Thiele (2005)). The efficiency of streamline simulation has led to its use for reservoir management (Lolomari et al. (2000); Thiele and Batycky (2006); Batycky et al. (2006)) and history matching (Wang and Kovscek (2000); Milliken et al. (2001); Caers et al. (2002)). Figure 1 displays an instantaneous streamline map for a two-dimensional case with three injectors and five producers.

A key difference between streamline simulation and other simulation approaches is that the two/threedimensional transport calculations are decomposed into a series of one-dimensional problems along streamlines, and this generally leads to significant speedups and is advantageous in the context of an optimization approach. However, in this work we focus on the additional information supplied by the streamlines to help build a local, linear surrogate model to approximate the necessary derivatives to drive the optimization approach. Specifically, we make use of flux pattern (FP) maps<sup>1</sup> (Thiele and Batycky (2006)), which are simplified views of the injector/producer pairs captured by the streamlines (Figure 2). As with streamlines, the FP maps represent the connectivity between injectors and producers at that instant in time. Here for each connection in a FP map we make use of four sets of parameters identifying ratios of injected/produced fluids from this connection. This is because each connection simply represents the sum of the volume fluxes (in and out) of all the streamlines associated with that injector-producer pair. Thus, for each connection we can define:

$$R_{I_iP_j} = \frac{(q_w^I)_{ij}}{(q_w^I)_i}, \quad R_{P_jI_i} = \frac{(q_f^P)_{ji}}{(q_f^P)_j}, \quad E_{I_iP_j} = \frac{(q_o^P)_{ji}}{(q_f^P)_{ji}}, \quad C_{I_iP_j} = \frac{(q_f^P)_{ji}}{(q_w^I)_{ij}},$$

<sup>&</sup>lt;sup>1</sup>The FPmap is protected by US patent #6,519,531 and used here with permission.





*Figure 1* Instantaneous streamline map obtained from a streamline simulation for a two-dimensional heterogeneous case with three injectors and five producers. The streamlines are colored by injector/pro-ducer bundles.

where  $(q_w^I)_{ij}$  is the rate of water going to producer  $P_j$  from injector  $I_i$ , and  $(q_f^P)_{ji}$  and  $(q_o^P)_{ji}$  are respectively the rates of total fluid and oil at producer  $P_j$  due to injector  $I_i$ .

 $R_{I_iP_j}$  and  $R_{P_jI_i}$  are volume ratios of total fluids injected/produced between well pairs with respect to the injected/produced fluids at the wells. These ratios are equivalent to the well-known well allocation factors, except that now they are determined using streamlines (Thiele and Batycky (2006); Batycky et al. (2006)), and therefore change in time. We note that in general  $\sum_{j=1}^{N_P} R_{I_iP_j} < 1$  (and  $\sum_{i=1}^{N_I} R_{P_jI_i} < 1$ ) since not necessarily all the water injected at  $I_i$  can be associated with a producer (and not all the fluid produced at  $P_j$  can be associated with an injector). The injection efficiency of an injector-producer pair  $E_{I_iP_j}$  is equivalent to an oil cut except that it is a ratio of oil produced to water injected. It is noteworthy that the oil cut of an injector-producer pair  $E_{I_iP_j}$  is something that is generally not known in standard simulation approaches. Finally, for compressible fluids  $C_{I_iP_j}$  is the volume ratio of total fluids (in and out) associated with an injector-producer pair, and represents the well-known voidage replacement ratio. Again, having this information for each injector-producer pair is unusual. For any reservoir the parameter set

$$\{R_{I_iP_j}, R_{P_iI_j}, E_{I_iP_j}, C_{I_iP_j}: i = 1, 2, \dots, N_I, j = 1, 2, \dots, N_P\}$$

describes uniquely the FP map at any instant in time and therefore the connectivity among injectors and producers in the field. In Figure 2 we show the ratios associated to injector  $I_4$  for the flux pattern map derived from Figure 1

#### Flow Problem Linearization and Optimization Process

The central premise of our work is that we can use the FP map to 'linearize' the flow problem between time steps, and thus estimate derivatives using a single (streamline) simulation. Although the FP map depends on the well control setting (i.e.  $\mathbf{q}_w^I, \mathbf{q}_f^P$ ), we assume that it does not change significantly in a modest small neighborhood around a given control setting. In this work this neighborhood of  $(\mathbf{q}_w^I, \mathbf{q}_f^P)$  will be referred as the trust region and will be denoted by  $\mathcal{N}(\mathbf{q}_w^I, \mathbf{q}_f^P)$ . The assumption above is partially





*Figure 2* The left figure is the flux pattern map derived from the streamline map in Figure 1. The right figure shows the connections associated with injector  $I_4$  and related parameters.

justified by the fact that a) in real fields rate changes for management purposes are usually done in small increments, and b) geological connectivity tends to dampen changes in streamline configuration due to small rate changes. As a result, we assume that oil and water production rates  $(q_o^P)_j$  and  $(q_w^P)_j$  may be reasonably represented as a linear function of water injection rates  $(q_w^I)_i$  using the FP map:

$$(q_o^P)_j = \sum_{i=1}^{N_I} (q_o^P)_{ji} = \sum_{i=1}^{N_I} (q_w^I)_i R_{I_i P_j} C_{I_i P_j} E_{I_i P_j},$$
(2a)

$$(q_w^P)_j = \sum_{i=1}^{N_l} (q_w^P)_{ji} = \sum_{i=1}^{N_l} (q_w^I)_i R_{I_i P_j} C_{I_i P_j} (1 - E_{I_i P_j}).$$
(2b)

The relation above allows us to linearize the nonlinear constraint  $\mathscr{F}(\mathbf{q}_w^{\mathbf{I}}, \mathbf{q}_f^{\mathbf{P}}; \mathbf{x}) = \mathbf{q}_o^{\mathbf{P}}$  in the original short-term problem. We reiterate that for a given set of controls  $(\mathbf{q}_w^{\mathbf{I}})_0$  and  $(\mathbf{q}_f^{\mathbf{P}})_0$  for which the FP map is determined, the linearization is assumed to be valid only in the neighborhood  $\mathscr{N}((\mathbf{q}_w^{\mathbf{I}})_0, (\mathbf{q}_f^{\mathbf{P}})_0)$ . Also notice that the linearization implies the assumption that the perturbation of the control settings on producers is determined by the perturbation of the control settings on injectors under the assumption of a fixed FP map (since the fluid rate of a producer is determined by the injection rates of injectors; see (2a) and (2b)). We can also generate another linearization based on the same FP map determined by  $(\mathbf{q}_w^{\mathbf{I}})_0$  as follows:

$$(q_o^P)_j = \sum_{i=1}^{N_I} (q_o^P)_{ji} = \sum_{i=1}^{N_I} (q_f^P)_j R_{P_j I_i} E_{I_i P_j},$$
(3a)

$$(q_w^P)_j = \sum_{i=1}^{N_l} (q_w^P)_{ji} = \sum_{i=1}^{N_l} (q_f^P)_j R_{P_j I_i} (1 - E_{I_i P_j}).$$
(3b)

From the point of view of physics, both linearized forms are reasonable, since the principal assumption is that oil and water are being produced by a displacement process rather than by expansion. From the point of view of mathematical optimization, both linearizations provide (partial) local information around  $((\mathbf{q}_{w}^{I})_{0}, (\mathbf{q}_{f}^{P})_{0})$ . The alternating method is a general approach for this situation (Boyd et al. (2011)); it alternately uses a part of local information to search for the next trial step. We implement our method in this alternating manner, i.e. at consecutive trial control settings, we use different linearizations to approximate derivatives.

The linearization function (either of the two forms above) is denoted here by  $L_{\mathscr{F}}(\mathbf{q}_{w}^{\mathbf{I}}, \mathbf{q}_{f}^{\mathbf{P}})$  and by construction it satisfies  $((\mathbf{q}_{w}^{\mathbf{P}})_{0}, (\mathbf{q}_{o}^{\mathbf{P}})_{0}) = L_{\mathscr{F}}((\mathbf{q}_{w}^{\mathbf{I}})_{0}, (\mathbf{q}_{f}^{\mathbf{P}})_{0})$ . Thus, in the neighborhood of a current trial



control setting given by  $(\mathbf{q}_{\mathbf{w}}^{\mathbf{I}})_0$  and  $(\mathbf{q}_{\mathbf{f}}^{\mathbf{P}})_0$ , the short-term optimization problem (1a)-(1f) can be replaced by the following linear programming problem:

$$\underset{\mathbf{q}_{w}^{\mathbf{I}},\mathbf{q}_{\mathbf{f}}^{\mathbf{P}}\in\mathscr{N}\left(\left(\mathbf{q}_{w}^{\mathbf{I}}\right)_{0},\left(\mathbf{q}_{\mathbf{f}}^{\mathbf{P}}\right)_{0}\right)}{\text{maximize}} \left(r^{o}\sum_{j=1}^{N_{P}}\left(q_{o}^{P}\right)_{j}-c^{wp}\sum_{j=1}^{N_{P}}\left(q_{w}^{P}\right)_{j}-c^{wi}\sum_{i=1}^{N_{I}}\left(q_{w}^{I}\right)_{i}\right)\Delta t,$$
(4a)

subject to

$$\left(\mathbf{q}_{\mathbf{w}}^{\mathbf{P}}, \mathbf{q}_{\mathbf{o}}^{\mathbf{P}}\right) = L_{\mathscr{F}}\left(\mathbf{q}_{\mathbf{w}}^{\mathbf{I}}, \mathbf{q}_{\mathbf{f}}^{\mathbf{P}}\right),\tag{4b}$$

$$\sum_{i=1}^{N_I} \left( q_w^I \right)_i = u, \quad \sum_{j=1}^{N_P} \left( q_f^P \right)_i = Cu, \tag{4c}$$

$$(q_o^P)_j + (q_w^P)_j = (q_f^P)_j, \quad j = 1, 2, \dots, N_P,$$
 (4d)

$$L_i^I \le \left(q_w^I\right)_i \le U_i^I, \quad i = 1, 2, \dots, N_I, \tag{4e}$$

$$L_{i}^{P} \leq (q_{f}^{P})_{i} \leq U_{i}^{P}, \quad j = 1, 2, \dots, N_{P}.$$
 (4f)

The solution for (4a)-(4f) is sought only in the trust region. As a consequence, in order to estimate a solution for (1a)-(1f) by means of an approach based on FP maps, it appears that a number of iterations that involve problems similar to (4a)-(4f) are needed. To solve the convex optimization problem (4a)-(4f), we use CVX, a package for specifying and solving convex programs (Grant and Boyd (2011)). We reiterate here that our method alternately uses linearization forms (2a)-(2b) and (3a)-(3b) along the iteration steps.

In each iteration step, the solution to (4a)-(4f) is the next trial control setting in the next iteration step. Note that the linearization is constructed with only one streamline simulation, hence the number of simulations required for one optimization step is one, which is much less than what is required for other derivative-free methods (on the order of the number of optimization variables  $N_I + N_P$ ). Therefore, the streamline-based method presented here can be expected to be more efficient than many derivative-free methods assuming the problem at hand can be reasonably represented by streamline simulation.

The radius of the trust region (defined by the Euclidean norm here) is modified during optimization to reflect the accuracy of the linearized model. If the linearization  $L_{\mathscr{F}}$  approximates the real model  $\mathscr{F}$  well in the trust region, then the radius of the trust region can be increased. On the other hand, if the approximation is not acceptable in the complete trust region, then the radius of the trust region has to be reduced. Consistent with trust-region theory (Conn et al. (2000)), given the control setting  $(\mathbf{q}_w^I)_0$  and  $(\mathbf{q}_f^P)_0$  used to determine the FP map, we estimate the accuracy of the approximation at the next trial control setting  $((\mathbf{q}_w^I)_1, (\mathbf{q}_f^P)_1) \in \mathscr{N}(((\mathbf{q}_w^I)_0, (\mathbf{q}_f^P)_0))$  by

$$\theta = \frac{\mathscr{J}\left(L_{\mathscr{F}}\left(\left(\mathbf{q}_{\mathbf{w}}^{\mathbf{I}}\right)_{0}, \left(\mathbf{q}_{\mathbf{f}}^{\mathbf{P}}\right)_{0}\right)\right) - \mathscr{J}\left(L_{\mathscr{F}}\left(\left(\mathbf{q}_{\mathbf{w}}^{\mathbf{I}}\right)_{1}, \left(\mathbf{q}_{\mathbf{f}}^{\mathbf{P}}\right)_{1}\right)\right)}{\mathscr{J}\left(\mathscr{F}\left(\left(\mathbf{q}_{\mathbf{w}}^{\mathbf{I}}\right)_{0}, \left(\mathbf{q}_{\mathbf{f}}^{\mathbf{P}}\right)_{0}\right)\right) - \mathscr{J}\left(\mathscr{F}\left(\left(\mathbf{q}_{\mathbf{w}}^{\mathbf{I}}\right)_{1}, \left(\mathbf{q}_{\mathbf{f}}^{\mathbf{P}}\right)_{1}\right)\right)}\right)$$
(5)

where  $\mathscr{J}$  denotes the profit function of oil/water production rates (Equation 1a and 4a). If  $\theta$  is close to one, it indicates that the approximation is acceptable, otherwise it needs to be revised. It is important to note, as described in the description of the algorithm below, that the computation of  $\theta$  does not require any additional streamline simulation or evaluation of the linearized model.

The short-term optimization method proposed in this work is as follows:

- 1. Set an initial guess of the optimal control setting  $(\mathbf{q}_{\mathbf{w}}^{\mathbf{I}})_0$  and  $(\mathbf{q}_{\mathbf{f}}^{\mathbf{P}})_0$ , and of the initial radius *r* of the trust region<sup>2</sup>.
- 2. Run a streamline simulation with  $(\mathbf{q}_{\mathbf{w}}^{\mathbf{I}})_0$  and  $(\mathbf{q}_{\mathbf{f}}^{\mathbf{P}})_0$ , and determine the FP map.

 $<sup>^{2}</sup>$ We use a trust region radius smaller but on the same order of the radius of the original feasible region of problem (1a)-(1f)as the initial radius.



- 3. Use the FP map information from  $(\mathbf{q}_{\mathbf{w}}^{\mathbf{I}})_0$  and  $(\mathbf{q}_{\mathbf{f}}^{\mathbf{P}})_0$  to set-up the approximation and linearized problem (4a)-(4f). Solve for  $(\mathbf{q}_{\mathbf{w}}^{\mathbf{I}})_1$  and  $(\mathbf{q}_{\mathbf{f}}^{\mathbf{P}})_1$ .
- 4. Run streamline simulation with  $(\mathbf{q}_{\mathbf{w}}^{\mathbf{I}})_1$  and  $(\mathbf{q}_{\mathbf{f}}^{\mathbf{P}})_1$ , determine the FP map, and determine  $\theta$  from Equation (5).
- 5. If  $\theta > 0.5$ , update  $(\mathbf{q}_{\mathbf{w}}^{\mathbf{I}})_0$  and  $(\mathbf{q}_{\mathbf{f}}^{\mathbf{P}})_0$  as  $(\mathbf{q}_{\mathbf{w}}^{\mathbf{I}})_1$  and  $(\mathbf{q}_{\mathbf{f}}^{\mathbf{P}})_1$ .
- 6. If  $0.5 < \theta < 2$ , increase *r* by 1.5 times. If  $\theta > 4$  or  $\theta < 0.25$ , decrease *r* by 1/3.
- 7. If the change in the objective function value is less than a user-defined tolerance, return  $(\mathbf{q}_{\mathbf{w}}^{\mathbf{I}})_0$  and  $(\mathbf{q}_{\mathbf{f}}^{\mathbf{P}})_0$  as solution, otherwise go to 2.

The trust-region approach implemented in this work does not fully follow the basic trust-region optimization method (see e.g. Conn et al. (2000)) because the gradient of the linearized model  $L_{\mathscr{F}}$  at  $((\mathbf{q}_w^I)_0, (\mathbf{q}_f^P)_0)$  does not coincide with the gradient of the function  $\mathscr{F}$  at that point (the surrogate in the basic algorithm in Conn et al. (2000)). However, based on the fact that the FP map can be used in general as an acceptable approximation of streamline simulation for short-term production computations, we can expect that both gradients provide similar information, and that the trust-region algorithm presented here yields an effective and efficient short-term optimization approach (as can be seen later in the case examples reported in this paper).

#### Long-Term Reservoir Management

The long-term reservoir management problem seeks a sequence of control settings during a long time horizon (typically more than 5 years) which maximizes some performance metric such as the expected net present value (NPV) (see e.g. van Essen et al. (2011)). This problem is of great importance in mature fields where a sustainable operation or strategy is a must to guarantee a high production level if oil price rises in future. In this work, the performance metric is NPV, defined as a weighted sum of the profit in each period:

$$NPV = \sum_{k=1}^{N_T} \left( r_k^o \sum_{j=1}^{N_P} \left( q_o^P \right)_{j,k} - c_k^{wp} \sum_{j=1}^{N_P} \left( q_w^P \right)_{j,k} - c_k^{wi} \sum_{i=1}^{N_I} \left( q_w^I \right)_{i,k} \right) \Delta t_k (1+d)^{-k},$$
(6)

where d is the discount factor, and all the time-varying quantities have the subindex 'k' with respect to those that were considered fixed in the short-term NPV definition used in Equation (1a).

In the scenario where the field production efficiency decreases to a low level and the expected oil price does not increase, shutting down the field before the end of the time horizon may be a more profitable strategy. To include this case, the objective function of the long-term optimization problem is modified to include the highest NPV over the time horizon. After the highest NPV value over the time horizon is reached, the field exploitation is stopped to avoid reduction of NPV (notice that here we do not account for costs associated with shutting down the field).



We define the long-term optimization problem as

$$\max_{(\mathbf{q}_{w}^{\mathbf{I}})_{1}, (\mathbf{q}_{f}^{\mathbf{P}})_{1}, \dots, (\mathbf{q}_{w}^{\mathbf{I}})_{N_{T}}, (\mathbf{q}_{f}^{\mathbf{P}})_{N_{T}}} \max_{0 \le n_{T} \le N_{T}} \left\{ \sum_{k=1}^{n_{T}} \left( r_{k}^{o} \sum_{j=1}^{N_{P}} \left( q_{o}^{P} \right)_{j,k} - c_{k}^{wp} \sum_{j=1}^{N_{P}} \left( q_{w}^{P} \right)_{j,k} - c_{k}^{wi} \sum_{i=1}^{N_{I}} \left( q_{w}^{I} \right)_{i,k} \right) \Delta t_{k} (1+d)^{-k} \right\}, \quad (7a)$$

subject to

$$\mathscr{F}\left(\left(\mathbf{q}_{\mathbf{w}}^{\mathbf{I}}\right)_{k}, \left(\mathbf{q}_{\mathbf{f}}^{\mathbf{P}}\right)_{k}; \mathbf{x}_{k}\right) = \left(\mathbf{q}_{\mathbf{o}}^{\mathbf{P}}\right)_{k}, \quad k = 1, 2, \dots, N_{T},$$
(7b)

$$(q_o^P)_{j,k} + (q_w^P)_{j,k} = (q_f^P)_{j,k}, \quad k = 1, 2, \dots, N_T, \ j = 1, 2, \dots, N_P,$$
 (7c)

$$L_i^I \le \left(q_w^I\right)_{i,k} \le U_i^I, \quad k = 1, 2, \dots, N_T, \ i = 1, 2, \dots, N_I,$$
(7d)

$$L_j^P \le \left(q_f^P\right)_{j,k} \le U_j^P, \quad k = 1, 2, \dots, N_T, \ j = 1, 2, \dots, N_P,$$
(7e)

$$(1-\alpha)\left(q_{w}^{I}\right)_{i,k-1} \leq \left(q_{w}^{I}\right)_{i,k} \leq (1+\beta)\left(q_{w}^{I}\right)_{i,k-1}, \quad k = 1, 2, \dots, N_{T}, \ i = 1, 2, \dots, N_{I}, \quad (7f)$$

$$(1-\alpha)\left(q_{f}^{P}\right)_{j,k-1} \leq \left(q_{f}^{P}\right)_{j,k} \leq (1+\beta)\left(q_{f}^{P}\right)_{j,k-1}, \quad k = 1, 2, \dots, N_{T}, \ j = 1, 2, \dots, N_{P}, \ (7g)$$

where  $n_T$  is a discrete variable that allows field shut down.

The long-term optimization problem is very similar to the short-term problem. Although the reservoir flow equations in (7b) change along time due to different initial conditions, the same reservoir simulator is used for evaluating each  $\mathscr{F}(\cdot,\cdot;\mathbf{x}_k)$ . As in the short-term case, there are upper and lower bounds for the injection/production rates. In practice, however, the rate controls are not expected to vary significantly from one period to the next (production strategies with a marked volatile behavior very often are not the most attractive ones to implement). Thus, in the long-term optimization problem the rates are not allowed to decrease or increase by relative factors of  $\alpha$  or  $\beta$  between two consecutive short-term periods (see (7f) and (7g)).

In the rest of this section, we propose a way to simplify this problem, and solve it as a sequence of simpler short-term problems. The streamline-based method introduced in the previous section will be applied to each of these subproblems to accelerate the whole optimization process.

#### Two-Stage Decomposition

We propose to approximate the solution of the long-term optimization problem by decomposing it into a sequence of  $N_T$  short-term problems. The short-term problems are linked using the analytical decline model as presented in the next section that sets the field production/injection rates as (equality) constraints period by period. In turn, each of the subproblems are solved using the linearized model based on streamlines introduced above and can be solved efficiently as described in that section. We emphasize that the subproblems are coupled (in this way we capture the nonlinearities associated with the flooding process), since the reservoir history of control settings in earlier periods will affect reservoir state  $\mathbf{x}_k$  in later periods. The solution of the sequence of short-term optimization problems yields a production strategy that is near-optimal for each period. Since each period is constrained by an optimal field injection strategy as given by the decline model, we expect the solution to be close-to-optimal for the complete production time frame. It is important to note that the approximate model described next allows calibration. We exploit this feature after each sequence of short-term optimization problems to improve the long-term model and iteratively repeat this two-stage optimization procedure until convergence. The two-stage optimization procedure used in this work is illustrated in Figure 3 for the first two iterations (in the first iteration we assume that the long-term approximate model is not available yet, and that the sequence of field targets does not change over time).

#### Modified Exponential Decline Model

In this section we explain how to assign the total field injection target over  $N_T$  periods, and how to improve this assignment iteratively. The key point in the whole process is to approximate how the field oil rate declines with a given sequence of field injection targets. We assume that the decline in the field





**Figure 3** Connection between the master problem (top) and subproblems (bottom) for an example with 17 wells that is described later, for two consecutive iteration of the two-stage optimization algorithm. The red line represents the field injection constraints, while the color bars represents the well control settings (water injection and fluid production rates).

oil rate is mainly a function of the field injection assignments, and that its dependency on the detailed well settings is weak. Then we can introduce a decline model for the field oil rate which depends only on the field injection assignment, and has two calibration parameters.

The starting point is the well-known exponential decline model (Aronofsky et al. (1958); Li and Horne (2003)):

$$N(t) = N_0 e^{-\lambda t},\tag{8}$$

where N(t) represents the recoverable oil reserve at time t,  $N_0$  is the recoverable oil reserve at time t = 0 (when the long-term optimization starts), and  $\lambda$  is the decline factor related to the reservoir state and operation. The cumulative field oil production associated with (8) is  $N_0(1 - e^{-\lambda t})$ , and the field oil production rate is  $\lambda N_0 e^{-\lambda t}$ . In Figure 4 we illustrate via streamline simulation the validity of this model for a reservoir operated with production settings that do not change over time, showing that the relation between field oil production rate and recoverable oil in place is approximately linear.

The decline model above describes reservoir behavior under constant control settings. We would like to modify this model such that it can be used in scenarios where controls change over time. First, we observe that the oil production rate associated to the original exponential decline model is proportional to both the current recoverable oil reserve N(t) and to the decline factor  $\lambda$ . Indeed, that defines the constituent equation that defines (8), i.e.,  $-dN/dt = \lambda N(t)$ . A reasonable generalization of this model can be made by considering a time-dependent decline factor  $\lambda(t)$  proportional to the current field water injection u(t) and to N(t). The validity of this generalization is also illustrated in Figure 4. Then the original exponential decline model can be generalized as

$$N(t) = N_0 e^{-c \int_0^t u(s) \mathrm{d}s},$$
(9)





**Figure 4** In the top figure, we operate a reservoir with constant control settings and obtain via streamline-based simulation a relation between field oil production rate and recoverable oil in place that is approximately linear. In the bottom figure, we operate the same reservoir with constant control until the 1810<sup>th</sup> day. At this point we increase the fluid rates in all wells by the same percentage. The figure shows that it is reasonable to assume that field oil production rate is proportional to field injection rate.



where c > 0 is a calibration constant. The cumulative field oil production associated to (9) is  $N_0 \left(1 - e^{-\int_0^t cu(s)ds}\right)$ , and the corresponding oil production rate is  $cN_0u(t)e^{-\int_0^t cu(s)ds}$ .

If the field water injection u(t) is given as a piecewise constant function with associated sequence of rates in  $N_T$  periods  $\{u_1, \ldots, u_{N_T}\}$ , then the field oil production rate at the end of period k is  $\gamma u_k N_0 e^{-\gamma \sum_{m=1}^k u_m}$ , where  $\gamma > 0$  is also a calibration constant (in general different from c).

It is worthwhile mentioning that two production strategies with identical sequences of field water injection rates may in general present different injection and production rates for each well, and therefore, do not necessarily yield the same field oil production profiles. As a consequence, the values of  $N_0$  and  $\gamma$  in both situations are not expected to coincide (note that  $N_0$  represents the total oil recoverable, which also depends on the particular production profile considered). Hence, it makes sense, and it could be beneficial regarding the accuracy of the solutions obtained, to iteratively calibrate the modified decline model. In this work, the calibration is formulated as a least-squares minimization problem where the cost function refers to discrepancy in field oil produced with respect to streamline-based simulation, and the optimization variables are the parameters  $N_0$  and  $\gamma$ . The optimization globalSearch with gradient-based local optima solver fmincon). It should be noted that, since the calibration process is applied to a previously performed streamline-based simulation, the computational cost of this process (which implies the solving of a relatively simple optimization problem where the cost function is not expensive, and only two variables are optimized) is negligible when compared to the other procedures in the complete two-stage optimization algorithm.

Although the modified decline model is simple, its accuracy (after iterative calibration) for our optimization purposes can be more than satisfactory, as shown in the examples section. This is in part explained by the fact that model trends can be more important in optimization than model precision. Thus, an approximate model can be useful to quickly detect, for example, in which periods oil production has to be increased, and to quantify that production change in a rough manner. Iterative calibration helps to improve accuracy in the solution by fine-tuning the results obtained. In Figure 5 we illustrate the approximation quality of the modified decline model for the first reservoir studied in the next section. First, we can see that when the field injection profile is uniform ( $u_k = u$ ), the cumulative field oil production over slightly more than seven years (each period has 30 days) determined by streamline-based simulation (blue symbols) is almost identical to the forecast computed by the calibrated decline model (magenta symbols). If this same decline model (without further recalibration) is used for a different and nonuniform field injection strategy (in this case, the profile corresponds to the solution obtained at the end of the first iteration in the two-stage optimization algorithm) the difference between streamline-based simulation (red symbols) and the modified decline model (green symbols) is larger but still relatively small. We can expect that the error will be reduced with additional calibration.

Given a field water injection strategy  $\mathbf{u} = \{u_1, u_2, \dots, u_{N_T}\}$ , for period *k* we can write  $u_k = \sum_{i=1}^{N_I} (q_w^I)_{i,k} = \frac{1}{C} \sum_{j=1}^{N_P} (q_f^P)_{j,k}$ , where, as before, *C* is the voidage replacement ratio. On the other hand, the decline model gives  $\gamma u_k N_0 e^{-\gamma \sum_{m=1}^k u_m} = \sum_{j=1}^{N_P} (q_o^P)_{j,k}$ , and the field water produced can be determined by subtracting the field oil produced from  $u_k/C$ . With these considerations we can rewrite the long-term opti-



**Figure 5** Cumulative oil production obtained with uniformly assignment of field injection target (blue) and its regression curve with decline model (magenta), cumulative oil production with updated assignment estimated by decline model (green) and obtained after the solution of the sequence of short-term problem (red)

mization problem (7a)-(7g) as:

$$\underset{\mathbf{u}}{\text{maximize}} \qquad \max_{0 \le n_T \le N_T} \left\{ \sum_{k=1}^{N_T} \left( r_k^o \sum_{j=1}^{N_P} \left( q_o^P \right)_{j,k} - c_k^{wp} \sum_{j=1}^{N_P} \left( q_w^P \right)_{j,k} - c_k^{wi} \sum_{i=1}^{N_I} \left( q_w^I \right)_{i,k} \right) \Delta t_k (1+d)^{-k} \right\},$$
(10a)

et to 
$$\sum_{i=1}^{N_I} (q_w^I)_{i,k} = u_k, \quad \sum_{j=1}^{N_P} (q_f^P)_{j,k} = Cu_k, \quad k = 1, \dots, N_T,$$
 (10b)

$$\sum_{j=1}^{N_P} \left( q_o^P \right)_{j,k} = \gamma N_0 u_t e^{-\sum_{m=1}^{N_T} \gamma u_m}, \quad k = 1, \dots, N_T,$$
(10c)

$$(1-\alpha)u_{k-1} \le u_k \le (1+\beta)u_{k+1}, \quad k = 1, \dots, N_T.$$
 (10d)

We stress that, unlike in (7a)-(7g), computationally expensive simulations are not involved in this optimization problem. Furthermore, the number of optimization variables has been reduced with respect to (7a)-(7g) from  $N_T \times (N_I + N_P)$  to only  $N_T$  (in most application the number of optimization intervals  $N_T$ is on the order of several tens or a few hundreds; for example, in the cases described in the next section,  $N_T$  is equal to 30). This typically means a reduction in the dimension of the optimization space of one or even two orders of magnitude since it is common to have tens or even hundreds of wells in a single field. It is also important to notice that the complexity associated with the thorough exploration of an optimization space increases dramatically with the number of variables. As a consequence, when solving (10a)-(10d) one is less likely to be trapped in local solutions that are not satisfactory from the cost function point of view than in (7a)-(7g). In this work, (7a)-(7g) has been approached using MATLAB Global Optimization Toolbox (multi-start optimization function GlobalSearch with gradient-based local optima solver fmincon).

We now have all the components in the two-stage reservoir management optimization algorithm. The overall procedure is as follows (see Figure 6):

- 1. Set the initial sequence of field water rates (e.g. a uniform strategy, i.e.  $u_k = u$  for the  $N_T$  periods) with given well control settings. This initial sequence of field rates may also be determined, for example, by considering some operational constraints due to facilities.
- 2. Solve the collection of short-term problems (1a)-(1f) subject to the sequence of field water rates



obtained in step 1 (or 4) to determine control settings for each well.

- 3. Based on the streamline-based simulation performed in the previous step for the optimized configuration, fit by regression the parameters  $\gamma$  and  $N_0$  in the modified decline model.
- 4. Solve (10a)-(10d) to obtain an optimized sequence of field water rates.
- 5. If the norm of the difference between the new and the old sequences of field water rates is smaller than some predetermined tolerance, then terminate the two-stage optimization process, and return the optimized configuration for all the wells. Otherwise, update solutions and go to step 2.



*Figure 6* Flow chart of the complete two-stage optimization approach for long-term reservoir management.

#### **Case Study**

#### Short-Term Problem

We test our method on two field models (Figure 7). Both models are modified versions of real fields for confidential reasons. In Field 1, we make the assumption of fluid incompressibility, i.e. C = 1. There are 10 producers, 7 injectors and an aquifer in the field. In Field 2, we allow fluid compressibility ( $C \approx 1.02$ ). There are 71 producers and 64 injectors. Both fields are mature fields with declining oil rate.

We choose direct search (Kolda et al. (2003)) as a comparison to our method. Direct search is a derivative-free optimization method which converges to locally optimal solution. An important disadvantage of direct search is that the computational cost is high due to the large number of function evaluations usually required.

For each field, we generate 60 short-term cases to test the streamline-based methodology compared with direct search. In every case, the short-term period is 90 days. For Field 1, the field injection limit in every period is  $1.35 \times 10^6 \text{ m}^3 (= 15000 \text{ m}^3/\text{day} \times 90 \text{ days})$ . For Field 2, the field injection limit is  $3.01 \times 10^6 \text{ m}^3 (= 33476 \text{ m}^3/\text{day} \times 90 \text{ days})$ . The 60 cases are generated as follows,





*Figure 7* Field 1 (left) and Field 2 (right) used in this section. The permeability distribution is shown where red and blue corresponds to high and low permeability values, respectively.

- 1. The initial reservoir state  $\mathbf{x}_1$  is given.
- 2. For  $t = 1, 2, \dots, 30$ ,
  - (a) Solve for optimal control settings  $(\mathbf{q}_{\mathbf{w}}^{\mathbf{I}})_k, (\mathbf{q}_{\mathbf{f}}^{\mathbf{P}})_k$  with streamline-based method;
  - (b) Solve for optimal control settings  $(\mathbf{q}_{\mathbf{w}}^{\mathbf{I}})'_{k}, (\mathbf{q}_{\mathbf{f}}^{\mathbf{P}})'_{k}$  with direct search;
  - (c) Update reservoir state  $x_{t+1}$  with  $\left(q_w^I\right)_k, \left(q_f^P\right)_k$  and  $x_t$ .
- 3. For  $t = 1, 2, \dots, 30$ ,
  - (a) Solve for optimal control settings  $(\mathbf{q}_{\mathbf{w}}^{\mathbf{I}})_k, (\mathbf{q}_{\mathbf{f}}^{\mathbf{P}})_k$  with streamline-based method;
  - (b) Solve for optimal control settings  $(\mathbf{q}_{\mathbf{w}}^{\mathbf{I}})_{k}^{\prime}, (\mathbf{q}_{\mathbf{f}}^{\mathbf{P}})_{k}^{\prime}$  with direct search;
  - (c) Update reservoir state  $\mathbf{x}_{t+1}$  with  $(\mathbf{q}_{\mathbf{w}}^{\mathbf{I}})'_{k}, (\mathbf{q}_{\mathbf{f}}^{\mathbf{P}})'_{k}$  and  $\mathbf{x}_{t}$ .

Notice that the difference between the first 30 cases and the second 30 case lies in the data/solution used to update the reservoir state.

The oil price used here is 107 USD/bbl. The cost to inject water and the cost to separate and dispose water producted are both 5 USD/bbl.

For the 60 short-term problems corresponding to Field 1, we compare in Figure 8 the optimized objective function value (field oil production) and the number of function evaluations (simulations) required. In general oil production values obtained with both methods are similar. On average, the streamline-based method is slightly better than direct search by 2%. As expected, the streamline-based method requires much fewer function evaluations (81% less in average).

For the 60 short-term problems corresponding to Field 2, we compare in Figure 9 the objective function value (field oil production) and the number of function evaluations (simulations) required. Overall, and as for Field 1, the oil production obtained from both methods are comparable. On average, the streamline-based method is slightly better than direct search by 3%. Again, the streamline method requires significantly fewer function evaluations (97% less in average).





*Figure 8* Comparison of short-term oil production optimization (top) and the number of reservoir simulations required (bottom) for Field 1.



*Figure 9* Comparison of short-term oil production optimization (top) and the number of reservoir simulations required (bottom) for Field 2. Notice that in the bottom graph the scaling for the y-axis is not linear.



The computational efficiency of the streamline-based method is even more evident for Field 2 because the associated optimization problem has more variables. The ratio of CPU time used by the direct search and streamline-based method is on the same order as the number of wells  $N_I + N_P$ . This is consistent with the streamline-based method reducing the number of simulations required in each step from  $N_I + N_P$  to one.

It is worthwhile mentioning that the streamline-based method returns a higher oil production than direct search. This can be explained by the fact that both methods aim at locally optimal solutions, and as a consequence can be attracted to different optimal regions of the search space.

## Long-Term Problem

To test our approach for long-term optimization, we use the same data as in previous section (i.e. Field 1 and Field 2). We also account for oil price by considering three noticeably distinct expected oil price curves. The first one is constant at 107 USD/bbl and corresponds to the oil price in January 2012. The other two are two segments (about seven years long; i.e. 30 periods of three months each) selected from the crude oil monthly prices in the last 15 years (Figure 10). The first segment (Oil Price Curve 2) is from August 1999 to January 2007 when the main tendency for the oil price is to increase. The second segment (Oil Price Curve 3) is from August 2004 to January 2012 when the oil price fluctuates. We test three discount factors 0%, 5% and 10% per year on each field and each expected price curve. This gives a total 18 cases: 2 fields  $\times$  3 price curves  $\times$  3 discount factors.



Figure 10 Crude oil price from January 1999 to January 2012, and constant extension until July 2019.

In Field 1, there is an upper bound for field injection rate of  $3 \times 10^4 \text{ m}^3/\text{day}$  and a lower bound of  $5 \times 10^3 \text{ m}^3/\text{day}$ . In Field 2, the upper bound and the lower bound for the field injection rate are  $1.152 \times 10^4 \text{ m}^3/\text{day}$  and  $6.695 \times 10^3 \text{ m}^3/\text{day}$ , respectively. In both fields, the field injection rate can be increased/decreased by up to 20% between consecutive periods. As in the short-term problem, the cost to inject water and the cost to separate and dispose water produced are both 5 USD/bbl.

In Case 1, the problem is based on Field 1 and Oil Price Curve 1 with a discount factor 0%. We already illustrated in Figure 5 the first iteration of the process. The exponential decline model is fitted with respect to cumulative oil production, and the recoverable oil reserve is estimated at  $N_0 = 3.28 \times 10^7 \text{ m}^3$  with a coefficient  $\gamma = 3.09 \times 10^{-8} \text{ m}^{-3}$ . With these two values, we solve the simplified master problem



(10a)-(10d). The new sequence of field injection target is plotted in Figure 11. In this case, since the expected oil price is constant and there is no discount, the two-stage method returns a strategy that continuously operates the field at the highest field injection rate possible. Thereafter we estimate the cumulative oil production and cumulative NPV with the decline model under the new sequence of field injection targets, and simultaneously solve the collection of subproblems under this new sequence of field injection targets (see again Figure 5). The two-stage process converges to a final solution in three outer iterations. The cumulative path of NPV associated to the final solution is shown in Figure 12. This final optimized NPV is \$9.3 billion, and this is 74% higher than the result that corresponds to constant field injection targets. Figure 13 displays the flow rates of six typical injectors and six typical producers in Field 1.



*Figure 11* Field injection targets considered for Case 1: constant injection (CI; blue), 'simple strategy' (SS; green), and field injection strategy optimized by two-stage method (TS; red). In this case 'simple strategy' is the same as constant injection, so the blue line is superimposed by the green line.

We also compare the solution with a so called 'simple strategy'. The 'simple strategy' uses only the information from the oil price. It starts from a field injection strategy  $\bar{\mathbf{u}}$  which is proportional to oil price curve and with the average injection rate equal to default control settings on which the field was operated before the optimization. Since in general  $\bar{\mathbf{u}}$  does not necessarily satisfy the constraint (10d), we determine the closest strategy to  $\bar{\mathbf{u}}$  using the Euclidean norm such that this constraint is satisfied. In our experiments, we have observed that this strategy is better than a strategy with constant rates. In Case 1, this strategy coincides with a constant injection strategy since the oil price in this case is constant and the NPV does not include discount.

In Case 2 and 3, the scenario is as in Case 1, except that the discount factor (per year) is equal to 5% and 10% respectively. The resulting cumulative oil production and cumulative NPV are shown in Figure 14. The discount reduces the impact of later periods in the total NPV. In Case 2, the strategy obtained by the two-stage streamline-based approach terminates after the 28<sup>th</sup> period; in Case 3 the optimized strategy stops production after the 23<sup>th</sup> period. In other words, in both cases the field is shut-in before the end of full optimization period.

Cases 4 to 6 are based on Oil Price Curve 2 (oil price increasing in time) with discount factors equal 0%, 5% and 10% respectively. With these oil price curves, the optimal strategy consists in not injecting





**Figure 12** NPV obtained for Case 1: default control setting with no optimization (NO; black), constant injection targets (CI; blue), 'simple strategy' (SS; green) and two-stage streamline-based approach (TS; red). The horizontal dashed lines mark the maximal NPV that those strategies can reach; The vertical dashed lines mark the best shut-down time for those strategies.



*Figure 13* Flow rates of 6 producers (top two rows) and 6 injectors (bottom two rows) for Case 1 with optimized field injection strategy.





**Figure 14** Case 2 (top; 5% discount factor) and 3 (bottom; 5% discount factor): the left column displays constant injection targets (CI; blue), 'simple strategy' (SS; green) and the field injection strategy optimized by the two-stage streamline-based method (TS; red); the right column displays the corresponding NPV which is additionally compared with default control settings (NO; black).

much water at the beginning, so as to allow for extra production when the oil price is high. Figure 15 shows that the optimized field injection strategy has a similar trend as the oil price (and 'simple strategy') curve, but they yield significantly different NPVs. When the discount factor is 0% or 5% per year, the two-stage approach returns strategies that do not terminate before the last period allowed (suggesting that additional profit could be realized in the future), while a constant strategy and 'simple strategy' shut the field down earlier since the 'overproduction' in previous periods prevents the field from producing profitable oil when eventually the oil price is high. For a discount factor of 10%, all strategies terminate before the 8<sup>th</sup> period.

Note that both the two-stage streamline-based approach and the 'simple strategy' fail to obtain a better solution than the constant field injection rate in Case 6. Here the net present oil price is much smaller than in the other cases due to a relatively low oil price, to a large discount factor, and to low oil production. As a result, the field appears to be almost uneconomic (we have disregarded factors such as the well drilling cost, or the inclusion of facilities). Additionally, the accuracy of the decline model may not be satisfactory in this particular case. If the decline model is fitted using production rates before the shut-down time (the information used for the regression refers to only five periods), optimization not be accurate enough. If, on the other hand, the decline model is fitted using the production rates of the entire time horizon, the regression process will minimize the total discrepancy of the decline model that correspond to the production rates in all periods, and this will result in relatively large error in the initial periods, which are extremely relevant in this problem.





**Figure 15** Case 4 (top; 0% discount factor), 5 (middle; 5% discount factor) and 6 (bottom; 10% discount factor): the left column displays constant injection targets (CI; blue), 'simple strategy' (SS; green) and the field injection strategy optimized by the two-stage streamline-based method (TS; red); the right column displays the corresponding NPV which is additionally compared with default control settings (NO; black).



Case	1	2	3	4	5	6
Field	1	1	1	1	1	1
Oil Price Curve	1	1	1	2	2	2
Discount Factor (% per year)	0	5	10	0	5	10
NPV (billion dollar) - CI	5.36	4.38	3.59	0.21	0.18	0.16
NPV (billion dollar) - SS	5.36	5.26	5.14	0.26	0.12	0.11
NPV (billion dollar) - TS	9.31	7.47	6.17	0.89	0.52	0.07
Relative Improvement (%) - TS vs. CI	73.80	70.66	71.85	313.92	185.61	-53.52
Relative Improvement (%) - TS vs. SS	73.80	42.02	20.23	240.17	318.77	-31.61
Case	7	8	9	10	11	12
Field	1	1	1	2	2	2
Oil Price Curve	3	3	3	1	1	1
Discount Factor (% per year)	0	5	10	0	5	10
NPV (billion dollar) - CI	2.72	1.99	1.49	7.83	6.65	5.88
NPV (billion dollar) - SS	2.77	1.99	1.52	7.62	6.66	6.23
NPV (billion dollar) - TS	3.42	2.37	1.63	7.81	7.11	6.52
Relative Improvement (%) - TS vs. CI	25.78	19.33	9.60	-0.33	6.96	10.92
Relative Improvement (%) - TS vs. SS	23.66	19.53	7.07	2.45	6.69	4.62
Case	13	14	15	16	17	18
Field	2	2	2	2	2	2
Oil Price Curve	2	2	2	3	3	3
Discount Factor (% per year)	0	5	10	0	5	10
NPV (billion dollar) - CI	0.41	0.37	0.34	3.02	2.52	2.10
NPV (billion dollar) - SS	0.44	0.37	0.32	3.35	2.75	2.13
NPV (billion dollar) - TS	0.86	0.40	0.36	4.19	3.13	2.52
Relative Improvement (%) - TS vs. CI	109.70	7.81	5.45	38.83	24.19	19.94
Relative Improvement (%) - TS vs. SS	95.19	7.86	12.87	25.10	13.98	18.34

**Table 1** Long-term problem results. CI: constant injection, SS: 'simple strategy', TS: two-stage streamline-based approach

Cases 7 to 9 are based on Oil Price Curve 3 which fluctuates over time. The curve contains two peaks, one higher but short peak around the 15<sup>th</sup> period and the other slightly lower but longer peak around the 25<sup>th</sup> period. Figure 16 displays the field injection rate and corresponding cumulative NPV obtained by the two-stage approach as well as by a constant strategy and the 'simple strategy'. In Case 7 where the discount factor is 0%, the two-stage streamline-based strategy evolves similarly to the oil price curve. Although the oil price in later periods is not as high as the first peak price, it stays at a high level for longer time. In a field where control settings are required to change gradually, a stable price peak can be more favorable than a short peak with higher value. In Case 8, the discount factor is 5% per year, and the solution present lower injection targets in later periods since the discounted oil price in these periods is lower. In Case 9, the discount factor is 10% per year and the second peak in the oil price curve is almost ignored in the net present value. Therefore, the solution obtained by the two-stage streamline-based approach terminates after the first oil price peak, otherwise the cost exceeds revenue in later periods.

Case 10 to 18 are based on Field 2 with the same settings of oil price and discount factor as Case 1 to 9. The corresponding results are shown in Table 1.

## Discussion

It is important to underline that our work is limited to floods where the principle production mechanism is by the injection of water and/or gas as opposed to production from expansion. This because the





**Figure 16** Case 7 (top; 0% discount factor), 8 (middle; 5% discount factor) and 9 (bottom; 10% discount factor): the left column displays constant injection targets (CI; blue), 'simple strategy' (SS; green) and the field injection strategy optimized by the two-stage streamline-based method (TS; red); the right column displays the corresponding NPV which is additionally compared with default control settings (NO; black).

linearization of the short-term problem assumes that there is a direct response on oil production at offset producers if injection is modified at connected injectors. If compressibility is the main production mechanism, this would no longer hold and the Flux Pattern map would no longer be representative of the flood.

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In addition, the modified exponential decline model is a simplified model of the long-term behavior of the reservoir and will not be able to properly model all situations. This may be particularly true when the oil production rate curves does not decrease monotonically because the optimization is able to mobilize previously unswept oil that manifests itself by the breakthrough of an oil bank. We suggest a manual or automatic mechanism to detect this sort of behavior in practice and modify the model accordingly.

The long-term optimization is a strong function of the expected oil price and thus the accuracy of the oil price expectation is important. Our work does not take account for the risk associated with the possible deviations from the assumed oil price.

## Conclusion

In this work, we propose a new approach for the long-term optimization of (water) floods in brown fields where the objective is the long-term NPV given an expected oil price and known operation costs. We solve the long-term problem by decomposing it into two stages: a long-term stage (or master problem) and a series of short-term stages. The long-term stage assigns the field injection/production rate for each short-term period according to the trend of oil price and the oil recovery decline of the field that make up the forecast period (5-15 years). The number of short-term periods is at the discretion of the modeler, but are usually on the order of 3 to 6 months in length. The decline of the oil recovery over the long-term model is done via an analytical exponential decline model, making the master problem very efficient. The exponential decline model is re-calibrated at each (outer) iteration in order to account for its dependency on the solutions of the optimized short-term problems.

Each short-term period is optimized using a linearized problem that is valid within a trust region and is obtained from the streamlines and quantified by Flux Pattern maps. The solution of each short-term stage is the optimal well control settings for that period constrained to the field injection targets assigned by the master problem. As opposed to traditional gradient-based methods and direct search methods, our optimization step requires only one flow (streamline) simulation per short-term stage. Usually the computational cost is a number of simulations that are on the same order as the number of wells. Thus the streamline-based approach is computationally very efficient, especially in cases where the field contains a large number of injectors and producers. We present two field examples that demonstrate the applicability of our approach and show excellent agreement with a more exhaustive (and costly) global optimization approach.

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