ABSTRACT

In this work we present the implementation of a simplified 3-phase, 3-component PVT model for simulating miscible gas injection. The simplified PVT formulation uses a Langmuir-type expression to represent miscibility between oil and gas components as a function of overall oil composition, and allows to determine phase compositions explicitly.

We demonstrate the applicability of the simplified PVT model in the context of streamline-based flow simulation. We use a 3-step approach: first we use a one-dimensional, full-physics, multi-component finite-difference simulation to calibrate the simplified PVT model. We then use the calibrated PVT model to perform a comparison between a full-physics finite-difference simulator and a streamline-based simulator using a synthetic WAG example based on an upscaled SPE10 data set. Finally, we perform a full-field simulation comparison of Alpine, an Alaskan field currently undergoing WAG injection. The Alpine model is a complex reservoir with approximately 100 wells.

INTRODUCTION

Streamline simulation has enjoyed a significant resurgence in last few years. The interested reader is referred to recent publications [Baker et al. 2002, Thiele 2001, Batycky et al. 1997] and the references therein for a detailed discussion on the subject.

Streamlines have shown significant promise in the area of miscible gas injection and compositional simulation [Thiele et al. 1997]. These are traditionally difficult problems to solve using cell-based simulation techniques, because large fluid mobility contrasts between injected gas and resident oil in conjunction with strong permeability/porosity contrasts can lead to severe time step restrictions. Streamline simulation largely overcomes these problems by decomposing the three-dimensional (3D) transport problem into a series of independent, one-dimensional (1D) problems along streamlines. The geometry of the streamlines is a direct reflection of preferential flow paths resulting from reservoir heterogeneity, and the time-of-flight (TOF) along the streamlines is a reflection of how quickly composition fronts will travel in the reservoir. The 1D transport along each streamline can be solved efficiently allowing to capture the chromatographic separation of components as they travel from the injection well to the production well. Streamlines can be periodically updated to capture the changing mobility field as well as new wells coming on-line or old wells being shut-in.

Streamline simulation is generally considered a “reduced” physics approach, since the theory is framed by the assumption of fluid incompressibility, and dispersive phenomena such as capillary
pressure, transverse physical dispersion, and fluid expansion are neglected. Streamline-based flow simulation therefore offers a “first-order” approximation of how a reservoir might react to gas injection. While capillary pressure, physical dispersion, and fluid compressibility are important, in certain instances miscible gas injection projects are plagued by early gas breakthrough caused by reservoir heterogeneity interacting with fluid mobility. In such cases, reduced physics streamline-based flow simulation might offer a viable alternative to more traditional approaches, particularly when trying to evaluate the uncertainty associated with field development decisions.

A SIMPLIFIED PVT MODEL

To investigate the first-order impact of gas injection, we propose a simplified PVT model as an approximation to the more rigorous thermodynamic phase equilibrium calculations generally used in compositional flow models. In particular, we are interested in retaining maximum computational efficiency while capturing two essential features of a miscible WAG injection: (1) the decreasing density of the oleic phase with increasing gas composition in the liquid phase, and (2) the decreasing viscosity of the oleic phase with increasing gas composition in the liquid phase. These are first-order effects that can significantly affect the sweep efficiency of a miscible WAG injection. The more miscible a displacement is the less gas override might impact the sweep efficiency of the injected gas, although this is partially offset by the increased density difference between the chase water and the oleic phase. Channeling of free gas is reduced as the injected gas goes into solution. At the same time, the viscosity of the oleic phase is reduced leading to a more favorable mobility ratio between the chase water and oleic phase. Similar simplified PVT models have been proposed by other authors in conjunction with the simulation of viscous fingering and near-miscible displacements [Koval 1963, Todd and Longstaff 1972, Fayers 1998, Zhou et al. 1999].

The basic premise of the simplified PVT model is that miscibility—condensation of the gas component into the oleic phase—is controlled by a single parameter omega (\(\omega\)), which in turn is a function of the overall oil and gas mass fractions. In our model, miscibility is not assumed to be a function of pressure, but only of composition in that the extent of miscibility—how much gas of the gas component actually dissolves into the oleic phase—is governed by \(\omega\). If \(\omega=1\) then all the gas present resides in the oleic phase and the system is a two-phase (oleic + water phase) system. If \(\omega=0\) then all the gas present resides in the gas phase and the systems is three phase (oil/gas/water). For \(\omega\)-values between 0 and 1, the system is also a three-phase system (oleic/gas/water), except that the gas saturation is smaller than it would be in the pure immiscible model because some of the gas has dissolved into the oleic phase. Similarly, the oil saturation is larger due to the lighter density of the oleic phase.

Formulation

The basic idea is that the gas component is allowed to become miscible with the oleic phase and the extent of miscibility is governed by a parameter omega (\(\omega\)). This is expressed as

\[
\begin{align*}
m_{g,g} &= (1 - \omega) z_g m_t \\
m_{o,o} &= \omega z_g m_t
\end{align*}
\]

where
\( m_{g,g} \) is the mass of gas in the gas phase
\( m_{g,o} \) is the mass of gas in the oleic phase
\( z_g \) is the overall mass fraction of the gas component
\( m_t \) is the total mass

Notice that if \( \omega = 1 \) then all the gas resides in the gas phase and \( m_{g,o} = 0 \). Conversely, if \( \omega = 0 \) then all the gas is in the oil phase and \( m_{g,g} = 0 \). If the oil component is assumed to reside only in the oleic phase (no vaporization) and similarly the water is assumed to reside only in the water phase then the phase compositions for the oleic phase can be determined explicitly as

\[
x_g = \frac{\omega \varrho_g}{\omega \varrho_g + \varrho_o} ; x_o = \frac{\varrho_o}{\omega \varrho_g + \varrho_o}
\]  

(2)

With the compositions of the oleic phase known, the effective density of the oleic phase is calculated using an ideal mixing model

\[
\varrho_o = \left( x_o + x_g \right) / \left( x_g / \varrho_o^* + x_{gas} / \varrho_g^* \right)
\]

(3)

where \( \varrho_o^* \) and \( \varrho_g^* \) are the pure-component densities at the pressure of interest. Similarly, the effective oil viscosity is calculated using a quarter-power mixing rule

\[
\mu_o = \left( x_o + x_g \right) \left( x_o / \mu_o^{1/4} + x_{gas} / \mu_g^{1/4} \right)^4.
\]

(4)

Here, \( \mu_o^* \) and \( \mu_g^* \) are the pure-component viscosities at reservoir pressure.

**Omega as a Function of Composition**

As the amount of oil component decreases, \( \omega \) should tend to zero since gas can only dissolve if there is some oil present. In other words, as the oil component decreases so must the ability of the oil to absorb free gas. A functionality like this is easily approximated using a Langmuir-type expression for \( \omega \) as a function of a normalized overall oil composition

\[
z_o = \frac{z_o}{z_g + z_o} \forall (z_g + z_o) > 0
\]

(5)

and

\[
\omega = C \frac{\beta \overline{z}_o}{1 + \beta \overline{z}_o}
\]

(6)

where \( C \) and \( \beta \) are constants describing the shape of the curve. Note that Eq.1 bounds \( \omega \) between 0 and 1. Since \( C \) is simply a constant, it is more useful to express \( C \) using a reference value of \( \omega \)
at \( z_0 = 1 \). We call this value \( \omega^* \) (omega star) and its value can be larger than 1. This results in the following expression for \( \omega \)

\[
\omega = \text{MIN} \left( 1, \omega^* \frac{1 + \beta}{\beta} \frac{\beta z}{1 + \beta z} \right)
\]  

(7)

Figure 1 shows examples of the functionality of \( \omega \) for different values of \( \omega^* \) and \( \beta \).

**A ONE-DIMENSIONAL EXAMPLE**

We first consider a one-dimensional example and show how \( \omega^* \) and \( \beta \) can be calibrated to yield an approximate solution to a full compositional solution. The intent here is to demonstrate that once calibrated, the same 1D solution can be used along streamlines to obtain a reasonable approximation of a full 3D compositional solution.

**1D Solution**

We consider a 1D WAG solution of an 8 component system (N2-C1, CO2-C2, C3-C4, C5-C6, C7-C8, C9-C13, C14-C24, C25+). Injected and initial compositions are given in Appendix 1.

The WAG schedule is 2200 Mscf/DAY of gas followed by 2000 STB/day of water on a 6 month interval starting after two years of initial water injection. The full compositional solution obtained with a finite difference simulator [Coats et al. 1998] is shown in Fig.2. Notice that injection and production pressures are reasonably constant up to 15 years, after which there is a significant change in the average pressure of the system. This part of the production profile is not expected to be matched by an incompressible
assumption, since the blow-down of the last 5 years is dominated by the compressibility of the system.

The idea then is to find a pair \((\omega^*, \beta)\) that will match key features of the full compositional solutions, such as the fall-off in the oil rate, water breakthrough, and the increase/decrease in the phase rates due to the WAG cycling. To find a good match we ran a 1D solution using values of \(\omega^*\) ranging from 5 to 0.2 and values of \(\beta\) ranging from 0.1 to 100. The behavior of the various solutions is shown in Fig. 3. A good pair is identified as being \(\omega^*=0.8\) and \(\beta=10\).

**Figure 3: Sensitivity of production profiles as a function of \(\omega^*\) and \(\beta\). Top left \((\beta=0.1)\), top right \((\beta=1)\), bottom left \((\beta=10)\), and bottom right \((\beta=100)\).**

**THREE-DIMENSIONAL EXAMPLES**

**3D Solution-SPE 10**

With the 1D solution calibrated, a 3D solution can be attempted using streamline-based flow simulation. As a first test, we use an upscaled version of the SPE10 comparative solution problem [Christie and Blunt 2001] with the central injector on WAG cycle and the off-set producers on BHP. The model size is 30x110x3. Results for this run are shown in Fig. 4. The
full compositional and compressible reference solutions obtained with a finite-difference model [Coats et al 1998] is shown in black. The colored lines are the phase rates obtained using the previously calibrated 1D solution in a streamline-based reservoir simulator [3DSL, 2002]

In this case the simplified PVT model—even under the assumption of incompressibility—is able to correctly capture the signature of the WAG cycling. Wells P3 and P4 clearly show the impact of the WAG injection, while wells P1 and P2 show little impact.

Figure 4: Three-dimensional solution for a WAG injection scheme using the SPE10 comparative solution project. The black lines are obtained using a full compositional finite-difference solution. The colored lines are obtained using the calibrated 1D solution in a streamline-based reservoir simulator.

3D Full Field Example-The Alpine Field, Alaska

To test our simplified PVT model further, we consider the Alpine field located on the North Slope of Alaska. Details on the Alpine field are given by Redman [2002]. As in the synthetic SPE10 example, we are mainly concerned about matching results obtained via an equivalent, full-field, compositional simulation using finite differences.

Data Preparation: Going from a full-field, compositional model to a simplified PVT model under the assumption of incompressibility and streamline-based flow simulation requires some data manipulation. In particular, for this study we:
• Used average values of $B_o$, $B_g$, $B_w$, and $R_s$ in the streamline-based flow simulator. Thus, all insitu volumes are assumed to be incompressible. The only expansion occurs as fluids are taken to the surface.
• To ensure that the finite-difference model and the streamline model removed approximately the same downhole volumes on a well-by-well basis, the surface phase rates from the finite-difference simulator were used to calculate equivalent total downhole rates for the streamline simulator. All wells in the streamline simulator were then set to the total downhole rates calculated from the finite difference simulator.
• Since the streamline simulator operates under the strict assumption of incompressible flow, there must be exact voidage replacement. That was ensured by allowing “open” boundaries in the streamline model allowing just enough volume to enter or exit the model to make up the difference between injection and production wells.
• Time steps were lumped to take advantage of the ability of streamlines to take large timesteps. While the finite-difference simulator used a total of 2353 timesteps, the streamline simulator used 79 timesteps.

Results of the simulations are shown in Fig. 5. A good field match is obtained, despite the significant assumptions made in the PVT model. Matches of individual wells are equally good but show some variation as would be expected (Fig. 6). Nevertheless, the quality of the match is noteworthy given the assumptions made.

DISCUSSION

The PVT model is a significant simplification of the original compositional fluid description as is the assumption of incompressibility—one might therefore reasonably expect the much simpler, reduced streamline simulation model to fail. Why then are the results shown in Fig. 4 through Fig. 6 as good as they are? This is probably due to two main reasons:

1. Both 3D models are dominated by reservoir heterogeneity, with the majority of the drive energy coming from injected fluids replacing insitu fluids. Production by insitu volume
expansion is a second-order effect. Under these circumstances, streamline-based flow simulation is expected to perform well.

2. While the PVT description is important, the finite-difference models are probably not able to resolve the interaction of phase behavior and heterogeneity to a sufficient degree of accuracy to make the effect noticable. In other words, the effective behavior caused by the coarseness of the grid is similar to a simplified pseudo model as presented in this paper. Trying to have a more detailed FD solution with a grid resolution fine enough to resolve the phase behavior of the system would probably be beyond current computational limits.

Applications

The importance of simplified models as described here is that engineers are faced with planning development scenarios with significant data uncertainty. For miscible WAG injection schemes in particular, the uncertainty goes beyond the usual issues associated with reservoir heterogeneity. Questions such optimal WAG cycles, well spacing, relative permeability all add to dynamic uncertainties which in turn are compounded by uncertainties in geological description. Under such circumstances, simplified, computational efficient models as presented here offer an attractive solution allowing to establish upper and lower uncertainty bounds for development scenarios.

EXTENSIONS

The PVT model is easily extendable to include vaporization of the oil component into the gas phase, by simply having an omega parameter for condensation of the gas component into the oil phase and omega parameter for vaporization of the oil component into the gas phase. Calling these parameters \( \omega_c \) and \( \omega_v \), we can write

\[
\begin{align*}
m_{g,g} &= (1 - \omega_c) z_g m_t ; \quad m_{g,o} = \omega_c z_g m_t \\
m_{o,o} &= (1 - \omega_v) z_o m_t ; \quad m_{o,g} = \omega_v z_o m_t
\end{align*}
\] (8)

where

- \( m_{g,g} \) is the mass of gas in the gas phase
- \( m_{g,o} \) is the mass of gas in the oleic phase
- \( m_{o,o} \) is the mass of oil in the oleic phase
- \( m_{o,g} \) is the mass of oil in the gas phase
- \( z_g, z_o \) are the overall mass fractions of gas and oil components
- \( m_t \) is the total mass

then the phase compositions are again trivially expressed as
With the phase compositions known, the effective phase properties are easily determined as per Eq.’s 3 and 4. Finally, both $\omega_c$ and $\omega_v$ can be made a function of composition such that condensation of the gas component and vaporization of the oil component are a function of how much of the other components is actually there. In other words, we are interested in having $\omega_c$ and $\omega_v$ tend to zero as oil component and the gas component go to zero, respectively. These expressions can be derived in the exact same way as Eq. (7). Finally, the model is trivially extended to multiple components by simply having a vaporizations/condensation value of omega for each component.

ACKNOWLEDGEMENTS

The authors would like to thank Ray Pierson for his assistance in running the finite difference models. The authors would also like to thank Phillips Petroleum, Phillips Alaska, Anadarko Petroleum, and StreamSim Technologies for permission to publish this paper.

REFERENCES


APPENDIX 1—PVT DATA

<table>
<thead>
<tr>
<th>CPT</th>
<th>MW</th>
<th>TC</th>
<th>PC</th>
<th>ZCRIT</th>
<th>AC</th>
<th>SHIFT</th>
<th>PCHOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>N2-C1</td>
<td>16.11</td>
<td>342.14</td>
<td>666.45</td>
<td>.2910</td>
<td>.0143</td>
<td>-.11830</td>
<td>70.51</td>
</tr>
<tr>
<td>CO2-C2</td>
<td>38.07</td>
<td>548.51</td>
<td>915.98</td>
<td>.2793</td>
<td>.1722</td>
<td>-.04402</td>
<td>133.10</td>
</tr>
<tr>
<td>C3-C4</td>
<td>48.71</td>
<td>695.37</td>
<td>596.75</td>
<td>.2761</td>
<td>.1668</td>
<td>-.07851</td>
<td>163.42</td>
</tr>
<tr>
<td>C5-C6</td>
<td>76.77</td>
<td>871.72</td>
<td>483.44</td>
<td>.2687</td>
<td>.2535</td>
<td>-.03034</td>
<td>243.39</td>
</tr>
<tr>
<td>C7-C8</td>
<td>101.79</td>
<td>1014.23</td>
<td>406.86</td>
<td>.2352</td>
<td>.3157</td>
<td>.08190</td>
<td>314.70</td>
</tr>
<tr>
<td>C9-C13</td>
<td>143.21</td>
<td>1149.59</td>
<td>329.85</td>
<td>.2495</td>
<td>.4725</td>
<td>.10833</td>
<td>432.75</td>
</tr>
<tr>
<td>C14-C24</td>
<td>245.87</td>
<td>1320.99</td>
<td>226.90</td>
<td>.2705</td>
<td>.6830</td>
<td>.07137</td>
<td>725.33</td>
</tr>
<tr>
<td>C25+</td>
<td>509.78</td>
<td>1600.85</td>
<td>188.19</td>
<td>.3258</td>
<td>1.0007</td>
<td>.04502</td>
<td>1477.47</td>
</tr>
<tr>
<td>BIN</td>
<td>.06293</td>
<td>.00999</td>
<td>.02222</td>
<td>.03443</td>
<td>.05003</td>
<td>.07793</td>
<td>.10111</td>
</tr>
<tr>
<td></td>
<td>.08464</td>
<td>.08249</td>
<td>.08564</td>
<td>.08531</td>
<td>.09471</td>
<td>.10021</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.00390</td>
<td>.01111</td>
<td>.02301</td>
<td>.04794</td>
<td>.05968</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>.00205</td>
<td>.00863</td>
<td>.02598</td>
<td>.03579</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>.00248</td>
<td>.01384</td>
<td>.03579</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>.00554</td>
<td>.03579</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>.04140</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Initial and Injected compositions for 1D calibration problem

<table>
<thead>
<tr>
<th></th>
<th>N2-C1</th>
<th>CO2-C2</th>
<th>C3-C4</th>
<th>C5-C6</th>
<th>C7-C8</th>
<th>C9-C13</th>
<th>C14-C24</th>
<th>C25+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>0.64980</td>
<td>0.12550</td>
<td>0.21160</td>
<td>0.01220</td>
<td>0.00090</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Injected</td>
<td>0.38448</td>
<td>0.06715</td>
<td>0.14194</td>
<td>0.06705</td>
<td>0.08994</td>
<td>0.10958</td>
<td>0.08782</td>
<td>0.05204</td>
</tr>
</tbody>
</table>