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Streamline Simulation

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ABSTRACT

Streamline-based flow simulation (SL) is now accepted as an effective and complementary technology to more traditional flow modeling approaches such as finite differences (FD). This is because streamline-based flow simulation is particularly effective in solving large, geologically complex and heterogeneous systems, where fluid flow is dictated by well positions and rates, rock properties (permeability, porosity, and fault distributions), fluid mobility (phase relative permeabilities and viscosities), and gravity. These are the class of problems more traditional modeling techniques have difficulties with. Capillary pressure effects and expansion-dominated flow mechanisms, on the other hand, are not modeled efficiently by streamlines.

Modern SL simulation rests on 6 key principles: (1) tracing three-dimensional (3D) streamlines in terms of time-of-flight (TOF); (2) recasting the mass conservation equations along streamlines in terms of TOF; (3) periodic updating of streamlines; (4) numerical 1D transport solutions along streamlines; (5) accounting for gravity effects using operator splitting; and (6) extension to compressible flow. These principles are reviewed here.

The application of SL simulation is presented in the context of what are generally considered important issues in reservoir engineering: (1) flood optimization; (2) history matching; (3) uncertainty in reservoir performance; (4) upscaling; (5) computational speed; and (6) miscible flooding. Novel, streamline-specific data is shown to add valuable engineering insight, as in the case of injector/producer efficiencies and as an aid in upscaling.

Finally, the outlook for streamline-based simulation is discussed in the context of compositional simulation, tracing streamlines through structurally complex geometries, fractured systems, and parallel computation. The speed and efficiency as well as the availability of new data make streamlines potentially the most significant tool for solving complex optimization problems related to history matching and optimal well placements.

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HISTORICAL CONTEXT & THE SIX KEY IDEAS

The current popularity of SL simulation should more aptly be termed a resurgence, given that streamlines—as pertaining to modeling subsurface fluid flow and transport—have been in the literature since Muskat and Wyckoff's 1934 paper and have received repeated attention since then. It is not the author's intention to give a full review of the technology in this paper. The interested reader is referred to the many papers and dissertations having extensive discussions and reference lists (*Crane et al., 2000; King and Datta-Gupta, 1998; Batycky et al., 1997*).



Figure 1: (Left) Streamlines and iso-potential lines for a direct line drive from Muskat and Wyckoff's 1934 paper "A Theoretical Analysis of Water-Flooding Network"; (Right) Streamlines derived using the steady state line source and sink solution for an infinite reservoir from B. Caudle's SPE Lecture Notes (1966) on Reservoir Engineering.

From the many different ideas and applications over the last 60 years, six key ideas have emerged as the basis for the current state-of-the-art of the technology.

Key Idea #1: Tracing Streamlines in Three Dimensions Using Time-of-Flight

One distinguishing feature of current SL simulation is that the streamlines are truly 3D, rather than 2D as in the streamtube methods of the 70's and 80's. While streamlines are generally depicted from a birds-eye perspective, streamlines now correctly account for the previously missing vertical component of the flow description and are therefore fundamental to the current success of the technology. From a practical point of view, the use of 3D streamlines no longer require geological models to be transformed into pseudo 2D areal models. Streamlines are truly 3D lines that can cut across simulation layers.



Figure 2:(Left) Although streamlines are generally depicted from a birds-eye perspective, (right) streamlines in modern SL simulation are truly 3D and correctly account for the previously missing vertical flow component.

The breakthrough work for tracing streamlines efficiently in 3D was that of Pollock (1988). Pollock's method is simple, analytical, and is formulated in terms of a time-of-flight (TOF)

coordinate. To apply Pollock's tracing method to any cell, the total flux in and out of each boundary is calculated using Darcy's Law. With the flux known, the algorithm centers on determining the exit point of a streamline and the time to exit given any entry point assuming a piece-wise linear approximation of the velocity field in each coordinate direction. The equations are simple: if v is the interstitial velocity ($v=u/\phi$), then a linear velocity description in the x-direction gives

$$v_x = v_{x0} + g_x(x - x_0);$$
 $g_x = \frac{v_{x\Delta x} - v_{x0}}{\Delta x}$

where v_{x0} is the x-velocity at x=x₀, and g_x is the velocity gradient in the x-direction. Since $v_x = dx/dt$, we can integrate the expression of the x-velocity (and in analogous fashion in the y- and z-direction) to get the exit times out of each face given an arbitrary entry point (x_i,y_i,z_i) and exit coordinates x_e, y_e, and z_e.

$$\Delta t_{x} = \frac{1}{g_{x}} \ln \left[\frac{v_{x0} + g_{x}(x_{e} - x_{0})}{v_{x0} + g_{x}(x_{i} - x_{0})} \right]; \quad \Delta t_{y} = \frac{1}{g_{y}} \ln \left[\frac{v_{y0} + g_{y}(y_{e} - y_{0})}{v_{y0} + g_{x}(y_{i} - y_{0})} \right]; \quad \Delta t_{z} = \frac{1}{g_{z}} \ln \left[\frac{v_{z0} + g_{z}(z_{e} - z_{0})}{v_{z0} + g_{z}(z_{i} - z_{0})} \right];$$

Since the streamline must exit from the face having the smallest travel time, $\Delta t_m = MIN(\Delta t_x, \Delta t_y, \Delta t_z)$, the exit locations are calculated by re-solving for x_e , y_e , and z_e using the minimum time:

$$x_{e} = \frac{1}{g_{x}} \ln[v_{xi} \exp(g_{x}\Delta t_{m}) - v_{x0}] + x_{0}$$
$$y_{e} = \frac{1}{g_{y}} \ln[v_{yi} \exp(g_{y}\Delta t_{m}) - v_{y0}] + y_{0}$$
$$z_{e} = \frac{1}{g_{z}} \ln[v_{zi} \exp(g_{z}\Delta t_{m}) - v_{z0}] + z_{0}$$



Figure 3: (Left) Pollock's 3D tracing method through a Cartesian cell. Given an arbitrary entry point, the time to exit and the exit point can be determined analytically (from Batycky et al. 1997). (Right) The exit point of one cell becomes the entry point for the next cell. By connecting exit and entry points a streamline is traced from injector to producer.

Pollock's equations are derived assuming orthogonal grid blocks, but very few real reservoirs models use such a strict Cartesian framework anymore. Using an isoparametric transformation, it

is possible to transform corner-point geometry grids (CPG) into unit cubes, apply Pollock's method, and then transform the exit coordinate back to physical space. Details of this transformation are given by Prevost et al. (2002) and by Cordes & Kinzelbach (1994). Using Pollock's method and modifications thereof it is possible to trace streamlines through any realistic grid used in reservoir simulation (*Prevost et al. 2002*).



Figure 4: (Left) Non-orthogonal cells can be transformed using an isoparametric transformation. Pollock's method can then be applied to the resulting unit cell (from Prevost et al. 2002). (Right) Streamlines in a homogenous 5-spot using a non-orthogonal grid and an isoparametric transformation (from Prevost et al. 2002).

With Pollok tracing algorithm, the natural extension of the 2D streamtube approaches of the 70's and 80's should be to define 3D streamtubes. But keeping track of geometrical objects in 3D is a cumbersome and numerically expensive proposition. A simpler and more efficient approach is to consider every streamline as the center of a streamtube whose volume is known— $\Delta V = Q_t \Delta \tau$, but the boundaries are not. Here Q_t is the total flow rate and $\Delta \tau$ is the delta TOF required to cross the gridblock. Thus, streamlines with small TOFs are equivalent to streamtubes with small volumes, i.e. fast flow regions. Conversely, streamlines with large TOFs are equivalent to streamtubes with large volumes, i.e. slower flow regions. Reformulating the transport problem along a streamline using TOF—rather than along a streamtube using volume—is the one key innovation that has allowed SL flow simulation to succeed for use in complex, 3D problems.

Key Idea #2: Recasting the mass conservation equations in terms of time-of-flight.

The understanding that using a TOF-variable along streamlines rather than a volume-variable along streamtubes came through the reformulation of the 3D mass conservation equations in terms of TOF. This was first shown by King et al. (1993) and later expanded on by Datta-Gupta and King (1995). The central assumption in the derivation was that the streamlines did not change over time—an assumption later relaxed as described in the next section. The derivation is simple (*Blunt et al., 1996, Batycky et al. 1997, Ingebrigtsen et al. 1999*). For incompressible and immiscible flow without gravity, the conservation equation for a phase j can be written as

$$\phi \frac{\partial S_j}{\partial t} + \vec{v}_t \cdot \nabla f_j = 0$$

where S_j is the saturation of phase j, $\vec{v}_t = \sum \vec{v}_j$ is the total velocity and f_f is the fractional flow of phase j. By defining a coordinate ξ that is parallel to v (i.e. a streamline) it is possible to write that

$$\vec{v}_t \cdot \nabla = \left| \vec{v}_t \right| \frac{\partial}{\partial \xi}$$

Now consider the definition of the TOF, which leads to the following expression

$$\tau = \int \frac{\phi}{\left|\vec{v}_{t}\right|} d\xi \quad \rightarrow \quad \frac{\partial \tau}{\partial \xi} = \frac{\phi}{\left|\vec{v}_{t}\right|} \quad \rightarrow \quad \left|\vec{v}_{t}\right| \frac{\partial}{\partial \xi} \equiv \vec{v}_{t} \cdot \nabla = \phi \frac{\partial}{\partial \tau}$$

allowing the three dimensional conservation equation to be re-written as

$$\frac{\partial S_{j}}{\partial t} + \frac{\partial f_{j}}{\partial \tau} = 0$$

There are a number of assumptions buried in this derivation. For example, that the flowrate along each streamline is constant, that the streamlines do not change over time and that the 1D solutions must have the same boundary and initial conditions as the 3D problem. But the derivation shows that a three-dimensional transport problem can be re-written in terms multiple, one-dimensional problems along streamlines. While this was known intuitively from the work on streamtubes, the TOF formulation offers a compelling mathematical framework. For the simple case of an incompressible waterflood it is thus possible to write

$$\phi \frac{\partial S_j}{\partial t} + \vec{u}_t \cdot \nabla f_j = 0 = \sum_{\text{streamlines}}^{\text{all}} \left(\frac{\partial S_j}{\partial t} + \frac{\partial f_i}{\partial \tau} \right)$$

The most important detail about this equation is that the total velocity in the 3D problem has disappeared into the TOF of each individual streamline. It is this decoupling of a 3D heterogeneous system into a series of 1D homogenous systems in terms of TOF that makes the SL method so attractive.



Figure 5: Filling a streamtube with a given volume is equivalent to walking the center streamline of the tube to a

Key Idea #3: Periodic updating of streamlines.

The fixed streamtube assumption was probably the single most significant drawback that prevented a wider use of the technology during the 70's and 80's. Martin & Wegner (1979) and Renard (1990) did consider changing streamlines, but it was not until the mid 90's that the fixed-streamline assumption was relaxed for good (*Thiele et al. 1996, Batycky et al. 1997*). While the interest at the time was to account for changing streamlines because of the rapidly changing mobility field in miscible gas injection problems, the real application was for problems with changing well conditions and gravity.

The idea was to treat the problem as a succession of steady-states by considering each updated streamline field valid only for a fixed time interval before updating it. The method worked well for mobility induced nonlinear problems, but mapping analytical, self-similar hyperbolic solutions (*Thiele et al. 1995, 1997*) would not allow to solve systems with changing well conditions and gravity, due to the requirement of uniform initial conditions along streamlines imposed by the analytical solutions. One additional element was needed: the ability to solve transport problems with generalized initial conditions along each streamlines (*Batycky et al. 1997*). Streamline

geometries could then change at will, guaranteeing that fluids would be transported in the correct directions by having initial compositions picked-up from the their position at the end of the previous timestep.



e, or wells being shut-in—as well as a changing mobility field. In general, wells will have a stronger impact on streamlines geometries that changes in the mobility field alone.

Key Idea #4: Numerical solutions along streamlines

Numerical 1D solutions along streamlines were first introduced by Bommer & Schechter (1979) to solve a Uranium leaching problem. Ironically, the streamlines were assumed fixed and the numerical solution was introduced because there was no analytical solution for the problem they were interested in. Batycky et al. (1997) now combined changing three-dimensional streamlines with a general, one-dimensional, numerical solution in TOF-space. This merging of ideas was instrumental in allowing streamline-based simulation to be used in real field cases, where streamlines would not only change due to mobility differences but also because of changing well conditions. With every new set of streamlines, the correct initial conditions could be mapped onto the streamlines—i.e. the conditions existing at the end of the previous timestep—and moved forward in time numerically. This allowed to move components correctly in 3D despite significant and radical changes in streamline geometries due to changing well conditions. Using 1D numerical solutions also made it possible to consider any 1D solution along streamlines, including complex compositional displacements (*Thiele et al. 1997*) or multi-component contaminant transport in aquifers (*Crane and Blunt 2000*).

One noteworthy issue arising from treating streamlines as discrete 1D systems is that the resulting discretization in TOF-space will result in a very irregular 1D grid, possibly with orders of magnitude differences between the smallest and largest cells. To maintain overall computational efficiency, it is imperative that a reasonable ratio between the smallest and largest cell along the cells be maintained. The approach used initially by Batycky et al. (1997) was to use a regular discretization of the original TOF grid along each streamline. Using sufficient amount of nodes (50-100) would guarantee an accurate representation of the solution, while attempting to minimize the mixing of fluids resulting from the regularization. More recent applications (3DSL 2003) have moved away from the regular grid in favor of an irregular grid that eliminated the smallest cells while aligning boundaries of the 1D cells with the original TOF grid. This approach avoids unnecessary mixing between cells while eliminating small cells that would limit the throughput of the 1D solver. Additional efficiency can be achieved using adaptive-implicit or fully-implicit schemes as wells a automatic mesh refinement. But even in these cases, elimination of very tiny cells might still be necessary.



Figure 7: The 1D time-of-flight grid along each streamline will be highly irregular for all but the most simple cases. Very small cells will co-exist with large cells thereby severely restricting the throughput of the 1D solver and reducing its efficiency. To eliminate the difference between small and large cells, the original TOF grid (red) can either be regularized (green) or left on an irregular spacing while binning smaller cells into larger ones (blue).

Key Idea #5: Gravity.

Including gravity presented a problem. The total velocity vector (which defines a streamline) is the sum of the phase vectors, but the phase vectors are not parallel in the presence of gravity. A solution was presented by Bratvedt et al. (1996) using the concept of operator splitting, an idea that had found previous application in front tracking (*Glimm et al. 1983, Bratvedt et al. 1992*). The concept of operator splitting in this case is simple, and revolves around solving the material balance equations in two steps: first a "convective step" is taken along the streamlines which is then followed by a "gravity" step along gravity lines—lines parallel to the gravity vector \vec{g} . In the gravity step fluids are segregated vertically according to their phase densities only. The simple conservation equation of incompressible, immiscible flow can be written as

$$\frac{\partial S_{j}}{\partial t} + \frac{\partial f_{j}}{\partial \tau} + \frac{1}{\phi} \frac{\partial G_{j}(S_{j})}{\partial z} = 0;$$

$$G_{j}(S_{j}) = f_{j} \sum_{i=1}^{n_{p}} \lambda_{i} (\rho_{i} - \rho_{j}), \quad f_{j} = \frac{|u_{j}|}{\sum |u_{i}|}$$

and is solved by splitting the conservation equation in two such that the solution of one becomes the initial conditions for the next.

$$\frac{\partial S_{j}}{\partial t} + \frac{\partial f_{j}}{\partial \tau} = 0; \quad \frac{\partial S_{j}}{\partial t} + \frac{1}{\phi} \frac{\partial G(S_{j})}{\partial z} = 0$$

While the order of the solution is mathematically immaterial, streamline-based simulation always solves the convective step first followed by the gravity step.



Figure 8: In the presence of gravity, the phase velocity vectors are not aligned with the total velocity vector defining the streamline. Thus, moving components along the total velocity for multiphase flow will not account for gravity segregation. This is corrected using an operator splitting approach.

Key Idea #6: Compressible Flow

All streamtube and streamline work in the past was restricted to the assumption of incompressible flow. The reason, of course, is that incompressible flow introduces simplifying assumptions that are particularly suitable for SL simulation. Two assumptions in particular are worth mentioning: 1) source and sinks correspond to wells, meaning that all streamlines must start in a source (an injector) and end in a sink (producer); and 2) the flow rate along each streamline (or streamtube) is constant. This second assumption is particularly important as it implies that transport along a streamline only involves solving for the component wave speeds, with each phase velocity simply given by $q_j = q_t \times f_j$. The problem is that there are no systems that are incompressible: all real field cases involve compressible flow characteristics. PVT properties can be a strong function of pressure, as in black-oil or gas condensate systems, and the voidage replacement ratios (reservoir volume in/reservoir volume out) can deviate significantly from unity, either locally or on a field wide basis leading to large pressure changes.

In compressible flow, streamlines can start or end in any gridblocks that act as a source or sink because of the compressible nature of the system, even if the block has no well. For example, in expansion type problems, any gridblock that sees its volume increase with decreasing pressure is a source and thus a potential starting point for a streamline. is a good example of this. It shows streamlines under primary depletion. Streamlines now start in the far field and end in producers that act as sinks, yet there are no sources (injection wells) in the traditional sense. In reality, the streamlines in have multiple sources along each streamline, since every gridblock a streamline passes through produces some fluid by expansion and therefore acts as a source.



Figure 9: Streamlines shown at two different times during primary depletion. In compressible flow, gridblocks act as sources even though there are no injections wells. A streamline will start in the far field and end in a producer collecting volumes from each gridblock it crosses.

Determining and tracing streamlines in compressible flow is not difficult. Pollock's tracing algorithm is valid regardless of how the flow velocities are determined. A significant extension to the mathematical formulation though is required to account for the coupling between saturations/compositions and pressure along the streamlines as well as accounting for a non-constant flow rate. One approach has been published by Ingebrigtsen et al (1999). A different, unpublished approach has been implemented into a commercially available code (3DSL 2003) and used for modeling compressible immiscible and miscible three-phase systems (Blackoil & Miscible Gas Injection). But while streamlines can model truly compressible systems, the inherent speed advantage over FD methods can diminish significantly depending on model size and governing displacement mechanisms. This is due simply to the constraint that if absolute pressure needs to be properly resolved to capture the transients, then limits on the global timestep size are very similar to FD methods. There are, however, still examples where compressible SL solutions are the only possible method to simulate very large secondary and tertiary displacement processes.

If PVT properties are only a weak function of pressure, the incompressible framework can still be used successfully through the introduction of "open" boundaries. Open boundaries can be distributed anywhere on the edges of the simulation model, with each boundary cell essentially on a

pressure boundary condition thus ensuring exact voidage replacement as required by the incompressible formulation. The approach works very well. Streamlines from the boundaries mimick the flow from the far field that would be observed in a closed but compressible system. Production profiles show similarly good comparison. The advantage with this approach is that historical injection/production volumes on a per well basis are honored exactly while the speed and efficiency of the incompressible formulation is retained.



Figure 10: Leaky boundaries allow to model systems with non-unit voidage replacement ratio and PVT properties that are a weak function of pressure in incompressible mode, while honoring historical injection/production well volumes .The coloring of the streamlines is based on TOF, with red showing smaller TOF than gray.

The previous six key ideas are central to the current state of SL simulation. Many mathematical details have been left out in the interest of time and clarity, but the curious reader will find many of the publications referenced to be excellent sources. For a comprehensive source of how streamline simulation came into being and many additional details, the reader is referred to Rod Batycky's indepth PhD Thesis (1997).

WHY STREAMLINE-BASED SIMULATION IS SUCCESSFUL

Many papers in the last few years have clearly shown the applicability of SL simulation for solving problems that have traditionally been difficult to model with more conventional techniques: near incompressible displacements in large, heterogeneous earth models. Rather than re-iterating the many excellent examples and important conclusions already in the literature, the questions as to why streamline-based simulation has been so successful and has so quickly re-surfaced as a powerful alternative to more classical simulation techniques is addressed here, and where possible, illustrated through examples. This question was also discussed by Baker et al. (2001).

Flow Visualization

Initially, the single most attractive feature for many engineers is the visual power of streamlines in outlining flow patterns. Rather than having to rely on visualization of saturations changes to reconstruct preferential flow paths, streamlines offer an immediate snapshot of the flow field clearly showing how wells, reservoir geometry, and reservoir heterogeneity interact to dictate where flow is coming from (injectors) and where flow is going to (producers). The ability to see the entire flow field at once is powerful and invariably yields unexpected and surprising flow

behavior of the model under consideration. Real fields, even those drilled in regular patterns, rarely show streamlines conforming to the expected distribution of fluids, and it is not unusual to see wells communicating with other wells far outside the expected pattern.

The visualization can be enhanced by considering different coloring schemes of the streamlines. Coloring by TOF (or drainage time DRT) with different cutoffs can vividly capture the growth of the swept/drainage area with time associated with injectors/producers.



Figure 11: Streamlines can capture in a visually appealing way how sweep/drainage areas associated with injectors (blue) and producers (green) grow with time (left to right). In this picture, the streamlines are kept fixed and are colored by TOF/DRT with an increasing time scale. The left most picture shows the volume associated with a TOF/DRT cutoff of 1 year, the right most picture the volume associated with a TOF/DRT cutoff of 20 years.

The connectivity of the reservoir that is expressed by the streamlines can be abstracted into a Flux Pattern Map (FPmap), which is probably what most reservoir engineers intuitively see existing behind the streamlines. The FPmap (*3DSL 2003*) is a schematic display of how injectors and producers are connected as a result of field rates, geological constraints, and flow physics, and is obtained by collapsing all the streamlines between a well pair into a single straight line connection. The FPmap is powerful because it highlights two central points all reservoir engineers are after: (a) what type of pattern flow exists in the field; and (b) what is the volumetric rate associated with an injector/producer pair. As shown later, the FPmap is an example of how streamlines are able to complement more traditional finite difference simulation by supplying novel information and lead to new and powerful reservoir engineering workflows, such as waterflood optimization.



Figure 12: (Left) The Flux Pattern Map (FPmap) is a powerful abstraction of information supplied by streamlines. (Right) It more easily shows the injector/producer pairs (green injector is supporting 8 producers) and line thickness' can be related to the flux between the well pairs.

Efficiency and Computational Speed

Computational speed and efficiency are often mentioned as one of the key advantages of SL simulation. The efficiency, though, comes at a price: simplified flow physics, a non-mass conservative formulation, and other simplifying assumptions. Nevertheless, for many real fields, the efficiency and speed of SL simulation offers the opportunity to solve outstanding engineering queries that can be addressed only with difficulty—if at all—using other approaches. The two most common examples are flow simulations on multi-million, geocellular models with complex heterogeneity, and simulations of hundreds of equiprobable realizations to asses parameter and prediction uncertainties. Why is SL simulation so efficient?

Memory Efficiency

There are two aspects to contribute to the memory efficiency in SL simulation:

- (1) streamline-based simulation is an IMPES-type formulation and therefore involves only the implicit solution for a single variable, pressure;
- (2) the tracing and the solution of the transport problem along each streamline is done sequentially, and thus only one streamline is kept in memory at any given time.

Combined with good coding practices, an efficient memory management of grid arrays, and an efficient linear solver such as Algebraic Multigrid (*Stüben 2000*), it is possible to run models that use approximately 400MB per million active cells (*Samier et al., 2002*). This means that it is possible to run fine-meshed models on relatively inexpensive computational platforms (PC's).

Computational Efficiency

Computational efficiency, on the other hand, is achieved because:

- (1) The 1D transport problem along each streamline can be solved efficiently.
- (2) The number of streamlines increases linearly with the number of active cells.
- (3) Streamlines only need to be updated infrequently (large time step sizes).

This gives rise to a near-linear scaling of run times with the number of active cells as shown in Figure 13.



Figure 13: Example of linear scaling of run time and number of streamlines as a function of active cells for the SPE Comparative Solution Project #10 using 3DSL, a commercial streamline simulator, on a PIII 866MHz PC.

While streamlines change over time due to mobility changes, gravity, and changing boundary conditions, for many practical problems, grouping well events into yearly or semi-yearly intervals and assuming that the streamlines remain unchanged over that period is reasonable. This is particularly true for mature waterfloods, an area where SL simulation works especially well. Field

simulations with 30 to 40 year histories are successfully and routinely simulated with 1-year time steps (*Baker et al. 2002*). In contrast to other simulation techniques, the size and number of the global time steps (frequency of streamline updates) is insensitive to the size and heterogeneity of the 3D model. This is an important detail that is often overlooked.

Solution of the mass transport along each streamline can be solved efficiently provided the 1D TOF-space is regularized or binned into an irregular grid (Figure 7). Proper discretization of the 1D problem is very important. Keeping small cells along the streamline—resulting from streamlines flowing through very high flow regions near wells or cutting cell corners, for example—would slow down the 1D transport solution along the streamline in much the same way as small cells tend to slow down IMPES solutions in regular simulators.

A good example to demonstrate the efficiency of SL simulation is Model 2 of the 10^{th} SPE comparative solution project (*Christie and Blunt, 2001*). The total run time, *T*, of any streamline simulation is approximately proportional to

$$T \propto \sum_{1}^{n_{ts}} \left(t^{solver} + \sum_{1}^{n_{sl}} t_j^{sl} \right)$$
 where

 n_{ts} number of time steps (number of streamline updates)

 t^{solver} time required to solve for the global pressure field (Ax = b) at each time step

- n_{sl} number of streamlines at each time step
- t_i^{sl} time to solve transport equation for each streamline

A near-linear scaling in computation time arises because:

- 1. The number of time steps (streamline updates) is independent of the model size, heterogeneity, and any other geometrical description of the 3D model. It is only a function of the number of well events and the displacement physics. For the SPE10 problem in Figure 13 all cases were run with the exact same number of streamline updates (24).
- 2. An efficient solver for solving the linear system of equations for pressure is expected to have a near-linear behavior as well (*Stüben 2000*).
- 3. The number of streamlines tend to increase linearly with the number of grid blocks all else being equal (Figure 13).
- 4. The time to solve the transport problem along each streamline can be made efficient by regridding the underlying TOF grid and choosing the number of nodes to use along each streamline regardless of the size of the underlying 3D grid.

The linear behavior with model size is the main reason why streamline simulation is so useful in the modeling of large systems. In FD's, finer models not only cause smaller timesteps due to smaller gridblocks but face additional timestep restrictions resulting from increased heterogeneity as finer models tend to have wider permeability and porosity distributions. The usual workaround for traditional simulation techniques is to use an implicit or adaptive-implicit formulation, but for large problems these solutions can become expensive, both in terms of CPU time and memory.

Full Field Modeling vs. Sector Modeling

A common practice when simulating large fields is to divide the field into sectors and simulate each sector individually. Streamlines, on the other hand, tend to suggest that fields rarely have clear patterns making it difficult to "carve" out a representative piece of the model that minimizes the flux across the simulation boundaries for all times. This problem is well known in the industry and the success of sector models strongly depends on properly estimating the flux in-and-out of the sector, or trying to use a border around the proposed sector to cushion the impact of the assumption.



Figure 14: Example of streamlines for a full field model (streamlines colored by injectors). Carving a sector out of a full-field model is a difficult problem that can be addressed by using streamlines to find the best no-flow boundaries in the reservoir over time.

The best approach, of course, is to model the entire field allowing for patterns to evolve to whatever is imposed by the interaction of well locations, well rates, reservoir architecture, and heterogeneity. But the ability to opt for a full-field model simulation requires an efficient simulation approach, both in terms of memory storage as well as computation time. Full-field models can get notoriously big (in terms of number of cells), even when using a limited number of gridblocks between wells. While streamline-based flow simulation makes some simplifying assumptions to achieve this efficiency, a full-field streamline model can be preferable to a sector model under traditional approaches, since the error introduced by choosing approximate sector boundaries can potentially be much larger and more significant than errors introduced by the streamline model itself. This is also an area where FD's and SL can complement each other nicely to achieve the common end-goal of better modeling the dynamic reservoir behavior. Since streamlines, by definition, are no-flow boundaries, a streamline map can be used to identify reasonable locations for sector boundaries (Figure 14). Although streamlines change with time, there might be delineations of the reservoir that standout as good lines along which to 'cut' the field into smaller, more manageable pieces.

Additionally, streamlines are able to give volumes associated with wells (Figure 15) by tagging gridblocks that are touched by all the streamlines ending in a predetermined well or group of wells. A novel approach to doing sector modeling then, is to take average volumes associated with wells of particular interest and simulate only that portion of the reservoir.



Flow Physics-Starting with the Simplest Model

There are simplifying assumptions in the numerical formulation of SL simulation, particularly with regard to flow physics. This is because the technique grew out of an incompressible framework, with the main modeling interest being to capture flow resulting from reservoir architecture and heterogeneity interacting with injected and produced volumes—problems which traditional FD simulators are not well suited for, particularly as the models become large (number of cells) and more heterogeneous (larger property contrasts). From the beginning then, the focus for streamlines was on determining displacement efficiencies, with run times increasing with increasing physics all else being equal. This is because physical complexity will tend to increase the number of streamline updates (n_{ts}) as well as the time required to solve the 1D transport problem along each streamline (t_{sl}), though it will not significantly impact the number of streamlines (n_{sl}).

Streamline simulation favors investigating problems starting with the simplest and fastest model and adding flow physics as required. A simulation study might, for example, start with inputting well locations, reservoir heterogeneity, reservoir architecture, produced and injected volumes and initially assuming (a) single phase, incompressible flow; then adding (b) fractional flow effects using phase relative permeabilities and phase viscosities; the next step being (c) including phase density differences; and finally (d) adding compressibility and more complex phase behavior. This natural progression of adding physical complexity is possible in traditional simulators, but is rarely done. Instead, the practice is to include as much physical complexity as the simulator allows, i.e. starting with the most complex model first, then unraveling the contribution of each component exactly the opposite of SL approaches.

igure 16 shows an example of an African offshore field that was in an exploratory planning stage. The geological model size was 30x140x245 and streamlines quickly indicated that gravity and vertical transmissibility barriers would be an important aspect in determining recovery for the field. For the geologists this was a satisfactory result since it captured the dynamic impact of their geological description. The reservoir engineers, on the other hand, were preoccupied with the issue of fluid description. For this example, simulations showed that a 3 phase model was required, though compressibility only seemed to have a second order effect. The ratio of run times for the various models was 1:4:7.3 (1Phase:3Phase:Blackoil). What is striking about this example is that an important conclusion about the flow behavior could be determined at the early modeling stage of the field (245 simulation layers), thus feeding back dynamic behavior of the model to the geoscientists. This was possible because simple models were constructed first and complexity added only as required to identify parameters having a first-order effect in the flow response.



igure 16: Streamlines offer rapid assessment of the impact of first-order flow effects on displacement efficiency. For this particular example, gravity and 3-phase flow are important aspects of the model that would have been missed using a faster but simpler single-phase flow formulation. Compressibility, on the other hand, is only of secondary importance.

Incompressibility and Well Controls

In truly incompressible systems, the absolute pressure level of the system is immaterial. All that is required is a pressure difference in order to calculate the total velocity using Darcy's Law. Though there are no real incompressible systems, the assumption of incompressibility is mathematically so powerful that it should be used whenever possible. For systems with strong water drives, systems having a voidage replacement ratio close to one, or systems that remain above bubble point, the assumption of incompressibility has been used with great success. These are also systems where streamlines work particularly well.

A very attractive consequence of incompressible systems is that historical well rates can be honored without having to previously ensure that the well models will give physical bottom-hole flowing pressures, i.e. P>0. This has important implications for history matching. Rather than starting the matching process by tuning well models to enforce historical volumes—in other words, trying to minimize the number of wells switching onto a pressure constraint—incompressible models allow the engineer to immediately begin matching the observed phase rates without regard to pressure. In fields where there are 100's, maybe 1000's of wells, it is practically impossible to honor historical well rates without allowing a good percentage of the wells to switch onto a bottom-hole-pressure constraint or even shut-in. Trying to fine-tune each well can be a painstakingly slow and costly exercise, which is made even more unnecessary in light of the fact that pressure might itself be of secondary importance in mature waterfloods. The ability to honor inter-pattern flow and generate novel, streamline-specific data (discussed further on), make history matching large, multi-well models a particularly well suited problem for SL simulation. The immediate consequences are overall better field and well matches obtained in significantly less time.

Novel Engineering Data

Streamlines go well beyond their visual appeal by producing new engineering data not available with conventional simulators. This is possibly the most interesting and valuable contribution of streamlines to the area of reservoir simulation, though the industry has not yet settled on how to best use this information. Since streamlines start at a source and end in a sink, it is possible, for example, to determine which injectors (or part of an aquifer) are supporting a particular producer, and exactly by how much. A high watercut in a producing well can therefore be traced back to specific injection wells or boundaries with water influx. Conversely, it is possible to determine just how much volume from a particular injection well is contributing to the producers it is supporting—particularly valuable information when trying to balance patterns.



Figure 17: Streamlines produce data not available from more traditional FD approaches such as reservoir volumes associated with individual wells (left), well allocation data between well pairs to calculated injection efficiencies (center), and reservoir connectivity between injector/producer pairs (right). All these combine to a powerful new view of the dynamic behavior of the reservoir.

Streamlines also identify reservoir volumes associated with any source or sink in the systems, since any block traversed by a streamline attached to a particular well will belong to its drainage volume. For the first time, it is possible to divide the reservoir into dynamically defined drainage zones attached to wells. All properties normally associated with reservoir volumes can now be expressed on a per-well basis, such as oil in place, water in place, and average grid properties. This data was not previously available and so there is little in the literature as to how to exploit it. One immediate use though is apparent: determining displacement and production "efficiency" on a well-by-well basis. This topic is covered in more detail in a separate section, but is surely one of the reasons for the keen interest currently existing for SL simulation.

APPLICATIONS OF STREAMLINE SIMULATION

Streamline simulation has made significant progress over the last 10 years and while the industry is still investigating and exploring the best use of this technology some key applications are emerging. These include waterflood optimization, history matching, and quantifying uncertainty in forecasting reservoir performance. Additional areas include, upscaling, ranking of geological models, and more recently using streamlines for improved gridding algorithms.

Waterflood Optimization

In many ways, the use of SL for waterflood optimization is a return to its roots. But the improvements in the technology allowing streamline to be used in complex, 3D geological models and with generic well controls opens up an entire new frontier for the application to real systems. The starting point for using SL's in flood optimization is the concept on an injection efficiency, which can be defined as:

$$I_{eff} = \frac{\text{off-set oil production [rb/day]}}{\text{water injection [rb/day]}}$$

Note the following about the above equation:

- There is an injection efficiency for each active injector in the field. The water injection rate is known (denominator), but the offset oil production (numerator) must be calculated using the information from the well allocation factors (WAF's), which in turn are calculated from the streamlines.
- It is possible to define an injection efficiency on an individual producer/injector pair. In this case, both water injection and offset oil production are computed from the streamlines.
- The injection efficiency defined as a ratio of rates is an instantaneous one. However, the equation also applies to cumulative volumes, in which case the result would be an average efficiency.

Although this discussion focuses exclusively on water injection and oil production, the definition of an efficiency can be extended to any type of injected and produced volumes by simply using the appropriate volumes in the definition of the injection efficiency. A simple pictorial illustration of injection efficiency is given in Figure 18.

This is a simple 100x100x1 heterogeneous grid with wells arranged in the classic five-spot pattern, but where flow is clearly unbalanced. What then is the instantaneous injection efficiency of injector I5? From the streamlines, I5 is connected to four producers: P3, P4, P5, and P7, and the injection rate of I5 is known. However, what is the offset oil production at the producers due to the injection of I5? Pictorially, this is simply the oil produced by the various streamlines bundles that start in I5 and end in the various producers, i.e. the oil produced by the red, green, orange, and

yellow streamlines. This data is easily tabulated as part of any streamline simulation and is referred to as well allocation factors (WAF). The information of the streamlines can also represented schematically using a Flux Pattern Map (3DSL 2003). A flux pattern map shows the connection between well pairs, and the thickness of the lines connecting the wells can be used to represent the strength of the flux between the pairs. Figure 18 shows that 48.5% of the injected volume is directed towards producer P5, whereas only 14.1% supports P7. The off-pattern producer P4 receives 16.7 of the injected volume of I5.



Figure 18: (Left) The injection efficiency of injector 15 is simply the oil produced at the offset producers divided by the injection rate of 15, where the production from the offset producers is simply the sum of the oil produced by the red, green, orange, and yellow streamline bundles. (Right) A Flux Pattern Map (FPmap) schematically depicts the data provided by streamlines, such as the injector-producer pairs and the strength of the connection. This figure shows the FPmap for injector 15 and its offset producers P3, P4, P5, and P7

Knowing the phase flow rates between well pairs it is then possible to automatically identify fluid cycling for optimization. The goal of a waterflooding optimization scheme is to maximize oil recovery for every barrel of water injected while honoring production/injection constraints, such as total available water, maximum injection/production rates, etc. The implementation though can differ in that either one (a) attempts to maintain an oil production plateau with less water, or (b) increase oil production due to better utilization of available water. Both goals can be achieved with aid of injection efficiencies. The first step in the workflow is to determine the injection efficiencies for each well, and to determine the average injection efficiency of the field. Injection efficiencies can be plotted in many ways and be color-coded for additional clarification. One example is to use a scatter plot as shown in Figure 19.





The dashed lines separate efficiency zones by increments of 25% and the unit slope line represents the limiting 100% efficiency line, i.e. the case in which every reservoir barrel of water injected produces a reservoir barrel of oil at the offset wells. With the individual injection well and field efficiencies determined, the central idea is to re-allocate water volumes by reducing water injection in low efficiency wells and increasing injection in high efficiency wells. "High" and "low" in this context are understood to be in relation to the average field efficiency. One reallocation scheme centers on the average field efficiency and smoothly increases or decreases rates depending on where they are compared to the average field efficiency (*Thiele et al. 2003*).

The above water optimization approach was applied to part of a giant Middle Eastern high energy, shallow marine carbonate reservoir The reservoir model consisted of a 214096 active cells, with approximately 230 injectors.



Figure 20: (Left) Injection efficiencies on Jan 1^{st} , 2003. (Right) Five year forecast using injection optimization every six months. Production constraints are accounted for and new injection wells come on-line in year 1 and 2. The dashed lines represent the reference "do-nothing" scenario in which all wells are left on the historical rates existing on 1/1/2003.

For simulation purposes, production history was lumped into 12 month intervals. Simulation of the 52-year period required 63 simulation time steps and approximately 90min on an 866MHz PC. Results of the optimization are shown in Figure 20. Note how a production is plateau is maintained while minimizing water production and using available water. The dashed lines represent the reference "do-nothing" scenario in which all produces and injectors are left on historical rates existing on 1/1/2003.

Pattern Balancing

Closely related to flood optimization is the ability of streamlines to aid in balancing patterns. By using the WAF's, the amount of injected fluid supporting any producer in the field is known exactly, and therefore the allocation of fluid between injectors and producers in a pattern is obtained automatically as part of any simulation. This also means that streamlines can immediately point out any fluid loss to wells outside a pattern—a potentially serious problem—and something empirical methodologies simply cannot capture. Figure 21 show a simple re-balancing of rates illustrated using streamlines and FPmaps.



Figure 21: Knowing the allocation of flow between wells and the visual display of streamlines allows patterns to be balanced more correctly and efficiently than with current techniques. From left to right: rates are progressively changed to yield a balanced pattern.

History Matching

Another promising application of streamlines is in the area of history matching. Because streamlines are tied to wells and at the same time delineate areas of the reservoir, the presumption must be that there is information hidden along streamlines that should help in the history matching process. This is indeed the case. Additionally, history matching generally requires multiple forward simulations. Having an efficient and fast method to generate reservoir responses is clearly a plus. There have been three main approaches to using SL's for history matching:

- 1. Derivation of analytical sensitivity coefficients which are then used to set-up an inverse problem (*Wen et al. 1998, Vasco et al. 1999*);
- 2. Defining average reservoir regions associated with wells and then changing grid properties in these regions to match production response (*Emanuel and Milliken 1997,1998*);
- 3. Modify grid properties traversed by streamlines to reduce/increase breakthrough times along these streamlines (*Wang and Kovscek 2000; Caers et al. 2001, Agarwal and Blunt, 2001*).

Deriving analytical sensitivity coefficient from streamlines is an efficient approach but assumes fixed streamline paths for all time (a linear model). Furthermore, there remains the problem of the solution of the inverse problem. Since streamlines are usually used in the context of large fields with many wells—i.e. those fields that are too computationally intensive for FD's—there remains a significant bottleneck in trying to solve such a large inverse problem.

More complementary to SL simulation are methods that attempt to use streamlines to directly modify gridblock properties. Avoiding the solution of an inverse problem, leaves the door open for history matching fields at a finer, geologically more realistic scale. Two methods stand out: the AHM approach of Emanuel and Milliken (1997,1998), and the work by Wang and Kovscek (2000), Caers et al.(2001) and Agarwal and Blunt (2001).



Figure 22: In the AHM approach streamlines are used to define average well zones, and wells are history matched by changing the statistics of the field—such as the Dysktra-Parson coefficient—but without attempting to change the spatial structures of the field.

The AHM approach of Emanuel and Milliken (1997,1998) has been applied with impressive success on a number of real fields. Its most distinguishing feature is that it does not rely on any mathematical algorithm to attempt convergence, and is much closer to traditional history matching forcing the engineer to use judgment and experience to modify model parameters. In the usual manner, the updated model is re-run and checked against field performance. The process is continued until an 'acceptable' match is achieved.

The basic premise of AHM is that a good match can be achieved by altering geological grid properties-permeability, porosity, net-to-gross ratio-associated with wells that need history matching. Since streamlines naturally define zones associated with wells, it is possible to implement such an algorithm using information provided by streamlines. Once zones associated with individual wells are identified, the statistics (but not the spatial structure) can be changed as guided by experience and understanding of the particular reservoir. Emanuel and Milliken (1997,1998) have identified several statistics that can be modified, the Dykstra-Parsons coefficient probably being the most important one. Figure 22 is a schematic of the approach: streamlines, possibly at different times, are used to define an average zone of the reservoir associated with a particular well (Figure 22-the average zone is for well P2). Once the average well zone is defined, grid properties for that zone can be manipulated to yield an improved history match. Important details remain. The AHM will not converge from a "disastrous" initial guess of the geological model-the premise is that a reasonable geological model does exists and that convergence can be achieved through sound reservoir engineering understanding of the process. All real fields will have changing well conditions, so that either an average zone has to be defined or there has to be lumping of well events. Additionally, there is nothing in the AHM approach that will maintain geological consistency, possibly one its major drawbacks.

In a similar fashion as the AHM method, the approach of Wang and Kovscek (2000) and Caers et al. (2002) tries to modify grid properties using information provided by streamlines. The basic idea here is to relate streamlines responsible for a history/simulation mismatch at the production wells to gridblock properties. In other words, the difference between measured and calculated fractional flow is related to an error in effective grid properties along certain streamlines linking an injector and a producer. The difficulty until know has been how to map the perturbation of the grid description of the model. In the method proposed by Caers et al. (2002), rather than mapping the perturbation directly onto the underlying numerical grid, the perturbation is used as a constraint via an iterative Monte Carlo technique to propose a new geological model, $k^{l}(\mathbf{u})$, that honors all the prior geological constraints. It is a Gauss-Markov method where each cell is visited randomly, and

an attempt is made to change each reservoir grid-block such that the new reservoir model better honors the streamline effective permeability constraint without destroying the geological continuity provided by the variogram.

Combined with multiple-point (mp) geostatistics (*Strebelle 2002*), the above approach can also overcome the limited information carried by the variogram about reservoir heterogeneity and curvilinear or strongly connected features important to flow. Combining mp-geostatistics with the perturbation of the probability distribution used to generate the static reservoir model opens the way to building geological models that are strongly constrained by the dynamic flow information provided by the streamlines yet remain geologically consistent and realistic. This is a significant achievement (*Caers et al. 2002*).



Figure 23: (Left) Mismatches between historical production data and simulated results can be linked to individual streamlines, and can be used to perturb individual cell properties (Middle) Yet this produces geological images that do not honor any of the original geological constraints. (Right) Using the streamline-induced property perturbations as a constraint, on the other hand, results in earth models that honor dynamic and geological constraints.

Ranking & Uncertainty in Reservoir Performance

When streamlines re-appeared in the early nineties, the profound impact geological models could have on the quality of simulation results was generally understood. The proliferation of sophisticated algorithms to model structure, faults, and properties is a testament to that. One of the earliest applications perfectly tailored for low-physics, high-speed, SL simulations seemed to be the ranking of fine-scale geological models—models that were clearly too large for traditional approaches but which needed ranking beyond the usual static variables such as hydrocarbon pore volume or connected volumes. Yet, ranking has been slow to emerged as one of the key applications of SL simulation. Why?

As pointed out by Gilman et al. (2002) the analysis for ranking and assessing uncertainty is not as straightforward as simply simulating a number of probable geological models. Complicating matters, uncertainty in future recovery can be as much a function of flow physics, well patterns, and well rates as it can be due to geological variability. Thus, the geoscientist must straddle with confidence two distinct disciplines: geomodeling and reservoir engineering. Analysis is further complicated by considering the economic component.

The dependence of rank on flow physics has been demonstrated via a very simple 3D, quarter-five spot example by Thiele and Batycky (2001). Figure 24 shows 30 equi-probable realizations in which the rank determined by TOF—linear, single phase flow where the TOF of the fastest streamline is taken as a cutoff for all other streamlines and the swept pore volume with TOF's less than the cutoff is used to rank the models—versus the recovery at breakthrough with increasing complexity of displacement physics. Figure 24 shows how quickly the ranking can change as flow physics is added to the simulation. Picking a P_{10} , P_{50} , and P_{90} based on TOF ranking is not a good

indicator of how recovery for a more complex displacement might behave. Unfortunately, single phase flow might not always be a good proxy for engineering more complicated enhanced or improved recovery mechanisms in a field. If geological variability is thrown into the mix as well, then the analysis becomes significantly more complicated. An efficient simulation approach alone—like SL simulation—is simply not sufficient to allow quantitatively useful analysis of uncertainty in reservoir performance. A statistical framework is needed that can channel the information into answers sought by engineers: which parameters contribute to the largest uncertainty? Can the uncertainty be reduced via additional data acquisition? What future reservoir management offers the best risk-return ratio?



Figure 24: Flow physics can significantly change the ranking behavior of systems of geological models. Here, 30 realizations are cross-plotted using the TOF-rank with the rank obtained using recovery at breakthrough with progressively more physics (from Thiele and Batycky, 2001).

The first difficulty in assessing uncertainty in reservoir performance is sampling the input parameter space. Consider the following simple scenario: a single, large geo-cellular modelmaybe having 2-5 million cells-is to be assessed for uncertainty in reservoir performance. Because the model is too large to simulate directly, a number of upscaled child-models are generated. Just three upscaling methods, two sets of relative permeabilities times, and five fault models lead to 90 scenarios, and the number is easily increased into the 1000's by considering just a few more additional parameters. Clearly, selecting models by a simple random Monte Carlo approach will not be successful if the number of parameters to be investigated is significant. A more intelligent sampling algorithm is required. One such approach has been suggested by Christie et al. (2002) which makes use of the Neighborhood Algorithm (NA), a stochastic sampling algorithm developed for earthquake seismology. The NA works by adaptively sampling the parameter space using geometrical properties of Voronoi cells to bias the sampling to regions of good data fit. The resulting ensemble of models represents a much smaller subset that can be used to predict the uncertainty in future performance using a tractable number of models, all of which are a good fit to past performance. Another alternative to sampling and evaluating the search space is to use evolutionary algorithms (Schulze-Riegert et al. 2002) which are characterized by only requiring the value of the objective function and do not need any gradient information. Evolutionary algorithms are robust and are inherently parallel, though convergence remains an outstanding issue and depends to a large extent to the amount of soft, expert knowledge provided by the user. Experimental design offers yet another approach to span the input parameter space in a systematic way so as to reduce the computational effort (White et al. 2001, Manceau et al. 2001).

Gilman et al. (2002) have pointed out the need for a more comprehensive statistical framework for assessing the impact of geological uncertainty on future performance. Regular well patterns—not necessarily representative of the actual well patterns or injected fluid rates existing in the field—can be used as a basis for ranking future performance. Because uniform patterns cover the entire

area of interest, and thus volumes of the reservoir away from known conditioning data, a more representative rank of the connectivity of the reservoir can be determined. In other words, the impact of the geological variability away from existing wells is emphasized here since, it is argued, that these areas are the ones will have a longer-term effect of performance and affect the decision of potential infill wells and recovery mechanisms.



Figure 25: (Left) Using a regular pattern of wells can be used to assess the impact of geological variability away from existing wells and the associated constraining data. (Right) A map of normalized standard deviation in oil recovery for all realizations can be used to show the largest uncertainty in oil recovery. (From Gilman et al. 2001)

Upscaling and Streamlines

Three SL features seem to be particularly useful for upscaling:

- 1. Producing fine-scale reference solutions (Samier et al., 2002).
- 2. Derive upscaled grid properties using streamlines (Christie and Clifford, 1997).
- 3. Lump cells using streamlines (Castellini et al. 2000, Portella and Hewett, 2000).

There are extensive variations on these themes in the literature which are not repeated here. The reader in encouraged to review the many excellent papers on this topic. In general, contributions can be separated into the application of streamlines as they pertain to validating upscaling methodologies, and application of streamlines as they pertain to actually generating average, upscaled properties.

In the area of validation, recent work by Samier et al. (2002) suggest that streamlines might offer an additional feature beyond simply generating fine-scale reference solutions against which to check upscaled solutions. The premise is that for upscaled system to have similar dynamic behavior as the original fine-scale model, wells should be draining similar volumes and those volumes should be connected in a similar way. Streamlines provide just that information. Comparing connected volumes over time for individual wells would offer a novel way to analyze the relative performance of upscaling algorithms.

Figure 26 and Figure 27 illustrate this idea using an example from Samier et al. (2002). Figure 26 shows that the streamline patterns of the upscaled models reproduce the fine-scale streamlines only in certain parts of the field. Figure 27 quantifies this discrepancy and compares it to the fractional flow behavior of the well. Well P2C is a good example where matching the well pore volume over time also leads to a good match of the water fractional flow. Well P3, on the other hand, has no match of the fine-scale pore volume, but the fractional flow is acceptable.



Figure 26: Streamlines colored by producers for two upscaled model and the reference fine-scale model. Good upscaling should produce similar streamlines patterns and volumes associated with individual wells between the fine-scale model and the upscaled models (from Samier et al. 2002).

An upscaling analysis based on well volumes and geometry might, in the end, be more relevant than simply comparing production profiles. For real field cases, it is well known that upscaling can eliminate smaller faults, transmissibility barriers, and other geometrical features so as to significantly change the flow pattern and drainage volume of a well. In such cases, there is no tweaking of flow parameters (such as relative permeability) that can remedy this problem, though a match with unphysical flow parameters might still be possible. The problem is one of flow geometry, and until the flow patterns in the upscaled model are not adequately matched to what is seen in the fine-scale model it is probably of little use to pursue approaches based on other parameters.



Figure 27: Streamlines allow to compare volume associated with wells between fine-scale and coarse-scale problems. Wells having good agreement on pore volume between fine-scale and coarse scale are likely to give better matches.

Miscible Flooding Using Streamlines

Streamlines are particularly powerful for modeling miscible gas injection and compositional simulation—these are the areas that revived the interest in streamline simulation in the first place (*Emanuel et al. 1989, Thiele et al. 1995*)—because the local sweep efficiency can be modeled accurately along each streamline and combined with the areal sweep efficiency given by the geometry of the streamlines themselves. In this way, a good estimate of the overall flood performance of the entire field can be determined (*Thiele et al. 2002*). Miscible and compositional flooding are traditionally difficult problems to solve using cell-based simulation techniques, because large fluid mobility contrasts between injected gas and resident oil in conjunction with strong permeability/porosity contrasts can lead to severe time step restrictions. The usual work around has then been to reduce the number of cells in the model at the expense of clouding the local displacement efficiency by numerical artifacts, carve out a sector of the field at expense of

misrepresenting the areal sweep efficiency, and/or use simplified PVT models (*Koval 1963, Todd and Longstaff 1972, Thiele et al. 2002*).

Streamline simulation is generally considered a "reduced" physics approach, since the theory is framed by the assumption of fluid incompressibility, and dispersive phenomena such as capillary pressure, transverse physical dispersion, and fluid expansion are neglected. Streamline-based flow simulation therefore offers a "first-order" approximation of how a reservoir might react to gas injection. While capillary pressure, physical dispersion, and fluid compressibility are important, in many instances miscible gas injection projects are plagued by early gas breakthrough caused by reservoir heterogeneity interacting with fluid mobility. In such cases, reduced physics SL simulation might offer a viable alternative to more traditional approaches, particularly when trying to evaluate the uncertainty associated with field development decisions.

In recent work, Thiele et al. (2002) used a simplified PVT model to calibrate the 1D displacement efficiency of an 8 component WAG system (N2-C1, CO2-C2, C3-C4, C5-C6, C7-C8, C9-C13, C14-C24, C25+), and then used that simplified 1D solution along streamlines to model a WAG simulation of the Alpine Field in Alaska. Results where then compared to a full-physics compositional simulator. The simulations compared favorably between the two approaches (Figure 28), demonstrating that under the appropriate circumstances, SL can be used to model complex process at a fraction of the computational cost of more traditional methods. The uncertainty in engineering WAG displacements goes beyond issues associated with reservoir heterogeneity; SL simulation can address more readily what optimal WAG cycles, well spacing to use, or relative permeability models to use, just to mention a few. This is a good example of SL simulation as a simplified, computational efficient model, yet able to establish credible upper and lower uncertainty bounds for future development scenarios of a complex field.



Figure 28: Field rate prediction for the Alpine Field, Alaska using a full physics compositional finitedifference model (solid) and a streamline model using a simplified PVT model (from Thiele et al. 2002).

MOVING FORWARD WITH STREAMLINE SIMULATION

Modern SL simulation is a powerful and complementary tool to traditional techniques used in engineering the upstream exploration and production of hydrocarbons. As a whole, the industry is still exploring the most optimal use of this technology and how it might be efficiently integrated into the current work flows used by individual companies and engineers. The next few years will bring a further maturing and extended application of the technology. It is not unreasonable to expect that most companies using conventional simulation technology today will, in one form or another, use SL simulation in their future work. What remains largely unknown is if new users groups, such as geologists and geophysicists, will adopt the technology in order to bring a dynamic

flow component to their analysis. What then might be expected over the next few years in terms of developments?

Compositional Simulation

The revival of modern SL simulation in the early 90's centered in part on miscible gas injection and as an alternative to the difficulties conventional methods had in resolving highly nonlinear, multiphase displacements through heterogeneous reservoirs. The numerical difficulties encountered by conventional methods for such problems continue to persist and remain a serious problem in many field-scale compositional studies. The only solution is usually to reduce the number of cells and/or reduce the geological complexity directly affecting the numerical performance. Representing the phase behavior via a more manageable number of pseudo components is the other alternative. In most cases, a combination of the two is used. But the price for these shortcuts is high. Numerical artifacts can become so severe as to completely mask any possibly beneficial phase behavior effects that might have been the reason for the numerical simulation in the first place. And going to more powerful hardware might not always yield the expected results, since run times for traditional simulation techniques can scale with a power of two or higher, quickly turning simulations into month-long computational marathons.

Streamlines remain unrivaled in the ability to efficiently transport components along flow paths, even in the presence of extreme permeability/porosity values. In the very specific case of multicomponent, multiphase flow where complex phase behavior is critical, the decoupling of the three-dimensional solution into a series of one-dimensional solutions is so attractive as a way to control numerical dispersion that it cannot be overlooked (*Thiele et al., 1997, Seto et al. 2003*). For large fields under gas injection, streamlines offer advantages that will unlikely be matched by traditional technologies: the ability to model full-field scenarios with reasonable inter-well spatial resolution in acceptable run times on affordable platforms.



Figure 29: Four-component displacement showing that numerical diffusion remains a serious problem for field-scale reservoir simulation, where the number of gridblocks between wells is often significantly less than 50 (blue line). From Thiele & Edwards (2001).

Tracing Streamline Through Structurally Complex Reservoirs

Tracing of streamlines currently rests on Pollock's bilinear interpolation, which in turn makes the fundamental assumption that there is a single velocity per cell face. Structurally complex reservoirs, on the other hand, often require multiple connections across a single face, as might be the case in the presence of faults. This adds a layer of complexity to the tracing algorithm, since cells might now have multiple velocities across a single face, theoretically even in opposite directions. Extension to tracing of streamlines in cells with multiple connections will give SL simulation the ability to model flow through heavily faulted and structurally complex systems while retaining much of its simplicity and speed. It will be an important extension to the technology and will likely allow its use for systems where traditionally more sophisticated meshing algorithm are used, such as finite-element or PEPI-grids. On-going research in this area should yield promising results within the next few years.

Streamlines in Fractured Systems

Streamlines have so far been primarily used in the context of single porosity systems, although intuitively fractured systems might seem suitable for streamlines since the fracture network can been interpreted as a fast flow conduit. But this does not address the matrix-fracture transfer, the main mechanism for production from fractured systems. Combining SL simulation with transfer functions—in essence source terms along each streamline—might make an extension to dual porosity systems possible. An important and necessary extension might also be the inclusion of capillary pressure, a mechanism not modeled in current streamline applications due to its diffusive behavior (capillary pressure acts in both directions, along and across streamlines, and therefore does not naturally lend itself to SL simulation). Such extensions are currently under review by several research groups and should yield an entire new range of applications for SL simulation if successful (*Di Donato et al. 2003*).

Simulation of Large System

Computationally, streamlines are processed serially and completely independent of each other. This architecture lends itself perfectly to parallelization. On a multiprocessor machine or cluster of machines, the transport problem along each streamline can be computed in parallel, thereby allowing significant speed-ups. Parallelization is required for the solution of large systems, systems that might be on the order of 10⁷ (or higher) gridblocks. Such systems are easily created in the case of giant oil fields (as might exist in the Middle East or Alaska) by simply allowing a reasonable block resolution between wells—25 grid blocks or more—and modeling the entire field without resorting to sector models. Even smaller reservoirs routinely have geomodels in range of 5 to 10 million cells. Large investments, both in terms hardware and software, have been made for the solution of such problems because it has been recognized that modeling the full field is critical for the engineering of optimal production scenarios. Streamlines are likely to play an important role in the future for tackling such difficult problems, particularly in assessing and screening potential scenarios for such giant models fields before resorting to more traditional approaches to confirm and cross-check a subset of the solution space.

New Work Flows

Although additional physics, such as compositional and dual porosity models, will clearly give SL simulation a larger field of applicability, the real breakthroughs for SL's are expected to come because of the integration of SL into existing and new work flows. The visualization, ease-of-use, and speed of SL's combined with novel engineering data are going to allow engineers to perform their day-to-day engineering work more efficiently and with better quality. Two of the workflows mentioned in this paper, waterflood optimization and history matching are but examples of what we can expect from SL simulation in the future.

THE FUTURE SIMULATOR

Reservoir simulation is a complex field and in most instances requires expert knowledge to obtain meaningful results. Graphical user interfaces and workflows help users along and provide guidance by shielding geoscientists from some of the complexities involved. Useful and efficient visualization of results are essential in the interpretation and validation of results. But it is a reality that the vast majority of users employ reservoir simulators as a black-box—as a transfer functions that takes a proposed reality and projects it into the future. There is little time to gain a deeper understanding of the solutions and how these are obtained. What is really desired are quick answers that can be obtained relatively easy. For the simulator of the future this means that it should not be limited to a single approach: it should ideally choose which simulation technique might be appropriate at what time domain and act accordingly—a "just-in-time" solution that is adequate for the solution that is currently sought. Intelligent simulators—i.e. simulators that will

automatically choose the most "efficient" solution technique for the problem at hand—will likely be the focus of future development in reservoir simulation. Streamline simulation is likely to be one of the techniques the intelligent simulator of the future will have at its disposal.

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